Problems from text

Problems 3.1.2, 3.2.5, 3.2.6, 3.3.3, 3.3.9.

Problem 1

Let \( x \mapsto \text{sign}(x) \) denote the sign function; i.e.,
\[
\text{sign}(x) := \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{if } x < 0.
\end{cases}
\]

Find the distribution of \( X \text{sign}(X) \) if \( X \sim N(0,1) \).

Problem 2

Let \( X \) be a random variable such that \( E|X| = \infty \). Define \( Y = |X - c| - |X| \), where \( c \) is a positive constant. Does \( EY \) exist? If so, find it.

Problem 3

Let \( X \sim N(0,1) \) and define
\[
Y = \begin{cases} 
X & \text{if } X > c \\
c & \text{if } X \leq c
\end{cases}
\]
to be the left-censored (or bottom-coded) version of \( X \), where \( c \) is a constant. Find the cdf and density of \( Y \). Calculate \( Eg(Y) \), where \( g \) is any function of \( Y \).

Problem 4

Use Markov’s inequality to show that the statement \( E X^2 = 0 \) implies that the random variable \( X \) is zero with probability one; i.e. \( \Pr(X = 0) = 1 \).

Problem 5

Use the result of the previous problem to show that if \( \text{var}(X) = 0 \), then \( X \) is constant with probability one; i.e. for some \( c \in \mathbb{R} \), \( \Pr(X = c) = 1 \).

Problem 6

Suppose we have the following distribution of students in a certain class:

\[
\begin{array}{|c|c|}
\hline 
\text{Category} & \text{Frequency} \\
\hline 
\text{Freshmen} & 28 \\
\text{Sophomores} & 25 \\
\text{Juniors} & 24 \\
\text{Seniors} & 23 \\
\hline 
\end{array}
\]

Is the median student in the class a Freshman, Sophomore, Junior or a Senior?

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Due: February 09, 2010.