QUANTIFICATION IN ENGLISH

SAMUEL C. WHEELER, III

I. Introduction

Two categories of constructions which we can call "degree words" and "number words", respectively, bear striking syntactic, semantic, and intuitive resemblances to constructions using "all" "some" and the other English constructions usually paraphrased by means of the existential and universal quantifiers. This paper argues that the universal and existential quantifiers are just particularly simple and orderly special cases of a group of two-place predicates of classes. The final section of the paper assesses the relation of this result to questions about logical form and logical truth.

That degree-word and number-word constructions are the same kind of construction as "all" and "some" constructions is indicated by certain syntactic phenomena and intuitions. We then show that a more powerful and coherent theory results from the identification. Number-words and degree-words can be substituted for the standard quantifiers salve congruitate. One raven may be black, many ravens may be clack, or all ravens may be black. If two ravens are black, then at least some ravens are black. Many who love Alma are loved by her, although not everyone who loves Alma is loved by her. Degree-words seem to be approximations of the standard quantifiers, in a certain sense, and "mean" the same sort of thing. The sentences: (1) "Some frogs are green", (2) "A few frogs are green", (3) "Many frogs are green", (4) "Almost all frogs are green", and (5) "All frogs are green", seem to mark areas on a continuum. Intuitively, "Almost all frogs are green" comes close to saying "All frogs are green" and says the same sort of thing.

A related point is that the number-words seem to be more precise versions of the degree-words. When we know there are many frogs and we want to know "how many", "thirty" is an answer. "Three frogs are green", and "Eleven thousand frogs are green" and "All but eleven frogs are green" amount to specifications of sentences (2), (3), and (4) above.
That degree-words and number-words are best construed as the same kind of word as the quantifiers is not a new view. The standard assimilation however, is to treat degree-words and number-words as operators on open sentences which yield closed sentences. But there are problems here.

Consider, for instance, the sentence "Many frogs are green". If we write this in a first-order theory as "\((Mx) \ (Fx \rightarrow Gx)\)" (where "M" is to be read as "most", analogous to "all") we obviously don't preserve truth. If we try "\((Mx) \ (Fx \& Gx)\)", we seem to come close, except that it seems the sentence would be true while the paraphrase would be false if not many things were frogs. "Many eagles have white heads" is true on the most common reading even though not many things are both eagles and white-headed. It's as if the quantifier attached itself in some way to the subject term of the sentence, adjusting its meaning according to how large the extension of the subject term is. However, a theory which takes "subject-terms" of quantified sentences seriously has difficulty with the fact that "Many eagles have white heads" is ambiguous. When there aren't many eagles, but, say, sixty percent of the eagles have white heads, "Many eagles have white heads" is true on one reading and false on another.

The degree-words seem to be quantifiers, yet the attempt to treat them as operators on a par with "\((Ex)\)" and "\((x)\)" seems to involve complications, not to mention expansions of the scope of the logical truths generated by the truth-definition for such a multiple-quantifier language.

The treatment of number-words as quantifiers involves other complications. Mathematical truth on that approach would have to be a matter of relations among an infinity of quantifiers. Possibly, quantifiers could get generated in much the same way complex predicates are generated in standard logics. But then the quantifier side of the truth-definition becomes as complex as the predicate side.

If degree-words and number-words are sufficiently quantifier-like that the assignment of completely different forms to constructions in which they occur and constructions in which "all" and "some" occur is counterintuitive, and their treatment in the manner of "\((Ex)\)" and "\((x)\)" is difficult, then a reasonable assimilation for a theorist to make is to treat "all" and "some" on a par with some well-founded theory of the degree words or number-words if one is available. Unless such a well-founded theory treated degree-words as operators on open sentences, this would amount to the thesis that quantifiers, construed as operators on open sentences which bind
QUANTIFICATION IN ENGLISH

their variables to form closed sentences, don’t occur in English deep structures. If degree-words and number-words and “all” and “some” have to be treated in the same way, and they cannot easily all be treated as standard “quantifiers”, then they should be treated in the way degree-words and number-words, prima facie, should be treated.

I don’t claim to have shown that a quantifier-multiplying theory, one which treats degree-words and number-words as operators on open sentences, can’t work. I only claim that a theory in which structure is relegated to predicates is prima facie simpler, and that since such a theory is workable and is further supported by tying in with an independently justified theory, it is preferable to quantifier-multiplying theories.

II. The Logical Form of Quantified Sentences in English

I begin by assuming the theory of degree-words developed in “Attributives and their Modifiers” and drawing some consequences from it. The theory proposes the following form for sentences like “John is much taller than Fred”:

“Tall (John, & (Tall(x, y(y = Fred))) & John e(x)(Tall(x, y(y = Fred)))”

“Much” is a construction which turns a comparative into an attributive. “Much taller than” can be read “Tall for someone who is taller than”. “Very” is a construction which operates on attributions and creates new attributions. “John is a very tall man” is to be read “John is tall for someone who is tall for a man and John is tall for a man”.

In the search for a unified theory of English, one notices that “much”, “very much” and “very” occur in sentences that seem to have very little to do with comparatives and attributives. For instance, we have “Much rice is brown”, “Very much rice is brown”, “Very many frogs are green”. Simplicity urges assimilation of these constructions to the constructions which occur with standard attributions and comparisons.

To begin the assimilation, I start with sentences which bear a strong intuitive resemblance to ordinary comparatives. I treat “rice”, a mass noun, as a singular term. I use a two-place relation, “Large” to express “More frogs than toads are green” by “Large(x(x is a frog and x is green), x(x = y(y is a toad and y is green)))” and to express “More rice than wheat is brown” by “Large(the brown rice, x(x = the brown wheat))”. These analyses yield comparatives to which “much” can apply, by the above analyses. “Many” is a variant of
"much" and is applied when the first argument of the comparison is a class.

For "many" to be the same construction in "Many frogs are green", as in "Many more frogs than toads are green", "Many frogs are green" must be an attributive formed by application of the construction to a comparative. The likely candidate for such application seems to be "Some frogs are green", which must therefore be construed as having the form of a comparative.

"Some frogs are green" and "Some rice is brown" are assigned the logical forms:

\[ L(\lambda)(Fx & Gx), (\gamma)(y = (\tau)(z \neq z)) \]
\[ L(\text{the brown rice}, (\gamma)(y = \text{the null individual})) \]

The first sentence says that the class of green frogs is large relative to the unit class of the null class, i.e. larger than the null class. The second sentence says that the brown rice is larger than the individual which overlaps nothing. "Many frogs are green" can then be analyzed as

\[ "L((\lambda)(Fx & Gx), (\gamma)(L(x, (\tau)(z = (\gamma)(y \neq y)))) \& ((\lambda)(Fx & Gx) e (\gamma)(L(x, (\tau)(z = (\gamma)(y \neq y))))"\]

The treatment of the ambiguity of "Many eagles have white heads" on this theory is that in the case where the sentence is true, the "reference" class of classes is a class of eagle-classes, for instance, while in the case where the sentence is false, the implicit reference class is the class of animal classes. The reduction of quantification-theory to set-theory is trivial, but has been rejected as theory on the grounds that set theory is less firmly grounded than quantification theory. Our argument is not just that such a reduction is possible, but that it is a correct account of how these constructions work in English. Assignments of forms to sentences which handle fragments of English adequately are not all equivalent — the ones which fit in better with the rest of the language are better analyses. The argument so far that "all" and "some" are two-place predicates depends on the fact that such an analysis allows a unified treatment of "many" and "much" and that such an analysis explains the felt resemblance between "Some frogs are green" and "Many frogs are green". The relation is just the same as that between "John is taller than Fred" and "John is much taller than Fred." A predicate is, as it were, iterated to form a stronger claim.

The scope-variations which provided an argument for treating "degree-words" as quantifiers in the first place are preserved on this
QUANTIFICATION IN ENGLISH

analysis. The predicates can apply to any pair of objects and “lesser and larger scope” of traditional quantifiers correspond exactly to how many imbedded class expressions the predicate dominates. To illustrate, we set out the narrow-wide-scope readings of “A few men kiss many girls” and “Many men kiss several girls,” leaving off membership clauses on constructed attributives. “A few men kiss several girls” can be read so that it has as a consequence that each of the few men kissed many girls (A) or so that it has as a consequence that each of the many girls was kissed by a few men (B).

(A) \( L(z)(Mz \& L((x)((Gx \& Kxz), (x)L(x)(z = (y)(y \neq y)))))) \)
\( (?)(L(z, (y)(y = (x)(x \neq x)))) \& L(z)(Mz \& L((x)(Gx \& Kxz), (x)L(x, (z)(z = (y)(y \neq y))))), (z)(z = (x)(x \neq x))) \)

The analysis of (A) consists of two conjuncts, the first claiming that the class of entities who are both men and such that they kiss many girls is not large for a class larger than the null class, and a second claiming that the same class of entities is large relative to the null class itself.

(B) \( L((x)(Gx \& -L((y)(My \& Kyx), (y)L(x, (z)(z = (y)(y \neq y)))))) \)
\( & L((y)(My \& Kyx), (z)(z = (x)(x \neq x))), (z)(z = (x)(x \neq x))) \)

(B) consists of a single attribution with a very complicated specification of the class which is said to be large for a class larger than the null class. This class consists of each entity which is a girl and such that the class of entities which kiss her and are male is not large for a class larger than the null class but is larger than the null class.

My primary argument that the form of quantified sentences in English is that of two-place predicates is that assimilations like the above are possible on this analysis and not on the standard analysis. Both analyses are adequate to the pre-analytic data of the restricted domain to which standard quantification-theory has applied itself. The above analysis not only manages this, but enables us to treat “very”, “many”, “much” and “all” in the same way in more of their occurrences in the language.

III. Numbers as Predicates of Classes

In “What Numbers Could Not Be” Paul Benacerraf first argues that applying all the criteria of choice it is rational to apply results in the selection of no particular sequence of sets from the candidate sequences of sets that might be identified with the natural numbers. Roughly, numbers couldn’t be all of several sequences of sets; there
is no reason to suppose that they are one rather than another; so they are not sets. We can call this the Argument from the Indeterminacy of Translation and the Falsity of Ontological Relativity. What the argument shows is that the attempt to identify numbers with sets meets with oversuccess, since what is true of numbers is true of many sequences of sets.

The obvious move for a theorist who doesn’t want his world expanded by countenancing both numbers and sets is to treat number-words as predicates of sets rather than as names of sets. A philosopher who wasn’t committed to finding out that 9 was an object or that mathematics was the science of relations among such objects would take occurrences of “nine” in sentences like “Nine frogs are green” as primary and treat the number-words as a system of predicates or functions or the class associated with a function from ordered n-tuples to truth-values. A sentence such as “$9 + 7 = 16$” where “9” seems to be a singular term would be treated as the generalized claim that for all ordered triples of classes the first two elements of which have no elements in common, if the first is a nine-class and the second is a seven-class, then, if the third is the union of the first two, it is a sixteen-class. Sentences “about numbers” would be qualified laws using number-predicates.

Benacerraf’s conclusion about the non-identity of the numbers with any sequence of sets goes well with this theory. Although there is a set of classes that are two-classes, there is no reason to identify the number two with that set. Given the systematic relations among number predicates, the same systematic relations will hold among the associated sets and among many other sequences of sets. But since there aren’t numbers any more than there is doghood, number-words don’t name any of these sets.

Benacerraf’s argument against theories of the above kind consists of pointing out that number-words, as they occur in natural languages, bear a closer relation to quantifiers than they seem to bear to either class-names or predicates. A generalized version of essentially his argument occurs in the first section of this paper. I draw a different conclusion from the resemblance. Rather than deciding that number-words are not predicates of classes, I take the argument to give evidence that numbers are quantifiers and thus predicates of classes, since there is independent evidence that quantifiers are predicates of classes.

The theory sketched below supports the theories about quantifiers advanced in Section II. Since numbers have traditionally been viewed as predicates of sets or sets of sets, treating quantifiers as
QUANTIFICATION IN ENGLISH

predicates of sets allows number-words to fit very well with linguistic data which make number-words appear quantifier-like. The neat fit developed below thus further strengthens the thesis that makes it possible, viz. that quantifiers are predicates of classes.

To sketch the theory of the logical form of number sentences in English, I list three of the sentences any adequate theory must account for.\(^3\)

(A) Three more frogs than toads are green.
(B) Three frogs are green.
(C) More than three frogs are green.

Sentence (A) has the form:

\[3((\exists)x(Fx \& Gx), (\forall)y(L(y, (\exists)z = (\exists)x(Tx \& Gx))))\]

This analysis rests on the view that the number-words are, as it were, specifications of the degree-words. A possible answer to "How many more \ldots?" is "Three more \ldots". For this reason, and in order to treat the "more" in "Three more frogs than toads are green" and "More frogs than toads are green" as non-homonymous, I treat "three" as taking the same kind of arguments as "many" and "much". That is, "Three" is a predicate of a pair of classes. Since "Zero" and "No" are number-words, there is no membership clause conjoined to the predication, unlike the case with "many" and "Much". Thus, "three" doesn't create an attributive construction, and cannot be operated on by "very".

For a class to be three, in relation to the class of classes that are larger than a given class, is for it to have three more members than the given class, to be "of third rank" among the classes that are larger than the given class.

The arguments for this interpretation are largely that it ties together the number-words with the other quantifiers and creates the fewest homonymies. That is, it seems that we could take "three" to be a two-place relation between a class and a given class, reading "Three frogs are green" as "The class of frogs is three in relation to the null class" and "Three more frogs than toads are green" as "The class of frogs is three in relation to the class of toads", but then the occurrence of "more" in "Three more frogs than toads are green" would be homonymous.

These accounts are not designed to turn mathematical truth into logical truth. The triviality of the explanation of what it takes for the "three"-relation to hold between a pair of classes amounts to taking "three" as a semantically primitive predicate. The theories using these predicates state their inter-relations, not the semantics of
the language in which they occur. Semantic isn’t physics or mathematics, and the semantics of mathematical language has suffered from thinking that it is.

“Three frogs are green” is analyzed in a way analogous to the analysis of “many frogs are green”:

$$3((x)(Fx \& Gx), (y)(L(y, (z)(z = (\forall)(x \neq x))))$$.

“More than three frogs are green” says that the class of green frogs is large relative to the class of classes that are three in relation to the classes that are larger than the null class:

$$L((x)(Fx \& Gx), (x)(3(x, (y)(L(y, (z)(z = (\forall)(w \neq ))))))$$.

The construction of the sentences “Many more than three frogs are green” and “Very many more than three frogs are green” is mechanical and obvious.

This theory does not pretend to have reduced mathematical truth to set theory. Relations among numbers will be mathematical truths, truths which depend on which predicate occurs in the sentence, not logical truths.

What the above shows, then, is that numbers can be treated as predicates of classes in a way which makes them analogous to the quantifier and degree-word-constructions to which they are pre-analytically similar. Semantic and syntactic evidence has been given to show that the analyses presented give the form of number-word sentences in English. In particular, the notion that number-words make precise claims replacing “many” sentences and “few” sentences, and the accommodation of the theory in a systematic way to sentences using units, make the theory plausible as a theory of English.

A final point that should be made about this theory is that it is not proposed as an improvement on mathematical language. It could well be that the system of mathematical description found in English is inferior to others that mathematicians have developed. The overall claim is just that the theory of English which results is better, not that the theory of mathematics is better or more convenient.

IV. Provability, Logical Truth, and Quantifier-Free Logic

This section sketches some consequences of the conclusion that constructions which have standardly been taken to be operators on open sentences are actually two-place predicates of sets. The conclusion that “any”, “some”, “each”, etc., constructions are predi-
cates of sets raises the following problems and questions among others:

1) According to the theory, the operation of set abstraction is primitive in English, which raises problems about non-referring singular terms and syntactic specifications of logical truth.

2) If "quantified" logical truths in English depend on truths using the "L"-relation and set-theory, then the logic of such constructions as "each", "all", etc., turns out to be formally weaker than on standard theories. The line between formal consequence and consequence relative to an empirical theory seems to be smudged by the predicate theory of "all" and "some".

3) Very generally, recent views about the relation of formal logic to natural languages have to be revised. Problems 1) and 2) shows that a return to the view that natural languages are logically and not merely notationally inferior to constructed languages is in order. Logic seems to be less an invariant than would be the case if every natural language was a first order (or higher) language with a complete logic.

Problem 1) arises when expressions such as "(x) (x≠x)" are formed with the class abstraction operator. Such an expression provably cannot have a reference, yet its English rendering, "the non-self-members", which might occur in "Some non-self-members are non-self-members", seems obviously a part of English. There are several ways a language which uses class abstraction as primitive can be treated, all of which picture the language as formally complicated and inelegant:

a) One solution paraphrases such expressions away be means of the existential quantifier. Such a tactic is, in principle, available even to theories which treat "all" and "some" as predicates. The theory wouldn’t give the usual English equivalents for "(Ex)", but would postulate a deep-structure "(Ex)" which doesn’t appear in surface structures. This theory would eschew any set-theoretic restrictions on set abstraction and treat set abstracts which cannot refer as still grammatical. Set theory, not grammar, is what tells us which set-abstracts refer.

The only problem I see with this theory is that it forces complications on the logical forms of the sentences it analyzes. Further, the reappearance of the existential quantifier amounts to a duplication of apparatus. Such a theory might be a good replacement or explication of English, but seems not to give the real form of these sentences.

b) The second solution resigns itself to the existence of non-referring singular terms and treats class abstracts as capable of yield-
aren't taken all the way back, though, since natural languages do have a logical form, albeit one which does not lend itself to mechanical determination of certain features. Thus, the relation of formal logic to theories of natural languages is both normative in somewhat the way Frege thought it was and descriptive in the way modern philosophers of language have come to realize it must be.

UNIVERSITY OF CONNECTICUT
STORRS, CONNECTICUT 06268
USA

NOTES

3. A more complete set of examples of sentences using number words is presented and analyzed in a longer version of this paper available from the author. The problem of how to generate complex number predicates in accordance with this analysis is dealt with therein.