ECONOMICS 6201 (FALL 2009)  
PROBLEM SET 3

XENIA MATSCHKE

PROBLEM A

A price-taking firm produces output $y$ using inputs $K$ and $L$ according to the production function $f(K, L) = K^\alpha L^{1-\alpha}$, where $\alpha \in (0, 1)$. Call the product price $p$ and the factor prices $r$ (rental rate) and $w$ (wage). Suppose that the firm maximizes revenue subject to the constraint that the expenditure for inputs cannot exceed a certain cash amount $C$.

1. What are the revenue-maximizing factor demands?
2. What is the revenue function $R(p, w, r, C)$?
3. According to the envelope theorem and using the general form for the production function, what is $\frac{\partial R}{\partial p}$? Prove this result from scratch.

PROBLEM B

Suppose you own a firm and have hired a production manager to run production for you. You have told him to produce 100 units of output each period in a cost-minimal way. After two periods, however, you have become suspicious whether the manager really minimizes costs. In period 1, you observed that input prices were $(w_1, w_2) = (2, 4)$ and the used factor quantities were $(x_1, x_2) = (4, 2)$. In period 2, you observed that input prices were $(w_1, w_2) = (3, 3)$ and the used factor quantities were $(x_1, x_2) = (3, 3)$. Is there any evidence that the manager did not minimize costs in period 1 or period 2?

PROBLEM C

Calculate the cost function for a general CES production function $f(x_1, x_2) = [ax_1^\rho + bx_2^\rho]^{\frac{1}{\rho}}$ using
1. the method seen in class,
2. the usual method (Lagrange function approach).

PROBLEM D

A firm has production function $f(x_1, x_2) = \min(x_1, 2x_2) + \min(3x_1, x_2)$.

1. What are the associated conditional factor demands?
2. What is the cost function?

PROBLEM E

A firm has production function $f(x_1, x_2) = \sqrt{3x_1 + 2x_2 + x_3}$ where factor $x_3 > 0$ is fixed in the short run. Calculate the short-run cost function. What are the variable costs, and what are the fixed costs? Also find the associated conditional factor demands.

---