PROBLEM A

Suppose two consumers live in an Edgeworth box economy, Ron (R) and Hermione (H). They consume all-flavor jelly beans (J) and magic chocolates (C). There utility functions are given as: 

\[ U_R = \min\{J_R, C_R\} \]

and

\[ U_H = \min\{J_H, \sqrt{C_H}\} \]

Ron is endowed with 30 jelly beans and 0 chocolates and Hermione is endowed with 0 jelly beans and 20 chocolates. Assuming that Ron and Hermione behave as price takers, what is the Walrasian equilibrium of this economy?

PROBLEM B

Consider a small country under autarky that produces manufacturing goods (M) and agricultural goods (F). The constant unit input requirements in terms of labor for manufacturing are 

\[ a_{LM} = 1 \] and \[ a_{LF} = 2 \] for food. The country’s labor endowment is \( \bar{L} \). The country’s representative consumer has a Cobb-Douglas utility function where manufactures have exponent \( \alpha = 0.6 \).

1. Normalizing the price of food to 1, find the autarky equilibrium for the country (consumption, production, relative price of good M, wage).
2. Now suppose that the country opens up to trade. Suppose the world market price of food is 2 and the world market price of manufactures is 4. Find the free trade equilibrium and compare it to the autarky equilibrium.

PROBLEM C

Consider a pure exchange economy with two brothers, Noel (N) and Linus (L), who consume two goods, \( x \) and \( y \). There is a total endowment of 1 unit of each good. Noel and Linus have identical utility functions of the form 

\[ U_i = \ln(x_i) + \ln(y_i), \]

where \( i = \{L, N\} \).

1. Calculate the Pareto set and draw it in an Edgeworth box diagram.
2. Suppose Noel’s and Linus’ mother distributes the endowments in such a way as to maximize the sum of Noel’s and Linus’ utilities. What is this allocation?

PROBLEM D

Alan and Ben live in an Edgeworth box economy. Each one of them is endowed with two goods. Call Alan’s endowment \( (\bar{X}_A, \bar{Y}_A) \) and Ben’s endowment \( (\bar{X}_B, \bar{Y}_B) \). In the beginning, the endowments are equally divided between Alan and Ben, so that \( (\bar{X}_A, \bar{Y}_A) = (\bar{X}_B, \bar{Y}_B) = (0.5\bar{X}, 0.5\bar{Y}) \). Alan’s utility function is 

\[ U_A = X_A^\alpha Y_A^{1-\alpha}, \]

and Ben’s utility function is 

\[ U_B = \beta \ln(X_B) + (1-\beta)\ln(Y_B). \]

Assume throughout that \( 0 < \alpha < \beta < 1 \), normalize the price of good \( Y \) to 1 and call \( p \) the price of good \( X \). Assume further that Alan and Ben both act as price takers.

1. Derive the Marshallian demands for Alan and Ben.
2. Find the market clearing price of good \( X \) and the equilibrium consumption quantities. Who will sell \( X \) and who will sell \( Y \) in the Walrasian equilibrium?
3. How does \( p \) change when Ben transfers one infinitesimal unit of his \( X \) endowment to Alan?
(4) Show using general (i.e. no particular functional form) indirect utility functions, that this transfer can reduce Alan’s utility in Walrasian equilibrium, and write down the sufficient condition for this to happen?

(5) Give an economic interpretation of how having more endowments can reduce utility. Is there a chance that the transfer reduces Alan’s utility in our example?

(6) Suppose that the transfer reduces Alan’s utility. First argue without any calculations that Ben’s utility must increase in this case. Then prove this result analytically.

**Problem E**

Consider a small open economy with two industries, \( X \) and \( Y \). The production functions are \( x = \frac{1}{100} \min(K_X, 2L_X) \) and \( y = \frac{1}{100} \min(2K_Y, L_Y) \). The country has factor endowments of \( \bar{K} = 450 \), \( \bar{L} = 300 \). It faces world market prices of \( p^W_X = p^W_Y = 1 \).

(1) Which industry is capital-intensive, which is labor-intensive? Explain.

(2) Find the equilibrium production quantities and the equilibrium factor prices.

(3) Carefully draw the Lerner-Pearce diagram for this economy.

**Problem F**

Alan (A) and Betty (B) live in Mansfield. Alan and Betty have income \( m \) and utility function \( U_i = C_i + \alpha_i \ln(G + 1) \), where \( C_i \) with \( i \in \{A, B\} \) denotes \( i \)'s consumption of a private good, and \( G \) is the public good consumption. Let \( 1 < \alpha_A < \alpha_B \) and \( m > \max(\alpha_A, \alpha_B) \) and assume that one unit of private good and one unit of public good both have a price of 1.

(1) Suppose the town of Mansfield knows the utility functions of Alan and Betty. Find the Lindahl equilibrium and show that it is efficient.

(2) Suppose that the town of Mansfield thinks about providing a certain size \( G_0 = e - 1 \) of the public good (\( e \) is Euler’s number). The town of Mansfield does not know the utility parameters \( \alpha_i \). It determines whether or not to build the public good and the payments by Alan and Betty as follows: It asks Alan and Betty for their respective \( \alpha_i \). If the reported parameters add up to more than \( e - 1 \), that is \( \hat{\alpha}_A + \hat{\alpha}_B \geq e - 1 \), \( G_0 \) is provided and both Alan and Betty pay half the cost, that is \( \frac{1}{2}(e - 1) \), otherwise, they pay nothing, plus the following additional payments for \( i \neq j \):

- If \( \hat{\alpha}_A + \hat{\alpha}_B \geq e - 1 \) and \( \hat{\alpha}_j \geq \frac{1}{2}(e - 1) \), consumer \( i \) pays no additional amount,
- if \( \hat{\alpha}_A + \hat{\alpha}_B \geq e - 1 \) and \( \hat{\alpha}_j < \frac{1}{2}(e - 1) \), consumer \( i \) pays an additional amount \( \frac{1}{2}(e - 1) - \hat{\alpha}_j \) to the town,
- if \( \hat{\alpha}_A + \hat{\alpha}_B < e - 1 \) and \( \hat{\alpha}_j \geq \frac{1}{2}(e - 1) \), consumer \( i \) pays an additional amount \( \hat{\alpha}_j - \frac{1}{2}(e - 1) \) to the town,
- if \( \hat{\alpha}_A + \hat{\alpha}_B < e - 1 \) and \( \hat{\alpha}_j < \frac{1}{2}(e - 1) \), consumer \( i \) pays no additional amount.

Show that with this mechanism in place, Alan does not have an incentive to lie about his true utility parameter \( \alpha_A \). (You don’t have to show this for Betty, although it is also true).