Varian problems

1.4, 1.10

Additional problem A

Consider the value function \( M(a) = \max_x \{ f(x, a); g(x, a) \geq 0 \} \). Suppose the objective function \( f(x, a) \) is concave in \((x, a)\) and that the constraint function \( g(x, a) \) is quasiconcave in \((x, a)\). Show that \( M(a) \) is concave in \( a \).

Additional problem B

Show that every convex function is quasiconvex, using the definition of quasiconvexity.

Additional problem C

Mark produces furniture \((F)\). For production, he uses wood \((W)\) and labor \((L)\) according to the production function \( F = f(W, L) \), where \( f \) is strictly quasiconcave. Mark hires workers at the local labor market for wage \( p_{L} \) per unit of labor, and he buys wood at the local lumberyard for price \( p_{W} \) per unit. Mark has a delivery contract with a distributor of his furniture to deliver \( \bar{F} \) furniture each period. Define the (minimum) cost function \( C \) as the minimum cost at which Mark can produce \( \bar{F} \), given his production technology and market prices.

1. Sketch Mark’s cost minimization problem in a \((W, L)\) diagram and label it. Where is the minimum cost in the picture? How does the strict quasiconcavity of \( f \) appear in the graph?
2. Write down the formula for the cost function \( C \) (for your answer, please look carefully at the definition given above). How do we call such a kind of function? Which variables does \( C \) have as its arguments?
3. Prove from scratch that \( \frac{\partial C}{\partial p_{W}} = W^{*} \) where \( W^{*} \) is the optimal (cost-minimizing) value of \( W \). Explain what the result means economically.

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