Consider the village of Lebanon, Connecticut, in the 18th century. A large towngreen is located right in the middle of the village, and the villagers use it to graze their cows. Cows can be bought at the farmers’ market in Coventry for $a$ dollars per animal. All villagers are allergic to milk, so they don’t consume it themselves, but rather sell it at the farmers’ market for $p$ dollars per gallon. The amount of milk in gallons that a cow produces depends on the amount of grass it can eat at the towngreen, and this in turn depends on the number of cows grazing there. Let us assume that the amount of milk produced (in gallons) by any one cow is given as \( \frac{100}{\sqrt{c}} \) where $c$ is the total number of cows.

(a) (10 points) Suppose each farmer (there are many of them) can afford at most one cow and let them decide on whether or not to buy the cow sequentially. Their decision depends on the average milk production of the existing cows and the price of cows. How many cows will be grazing on the towngreen in equilibrium? What will be the average profit per cow? How does this compare to the efficient number of cows and the profit per cow?

(b) (10 points) The farmers have come to the conclusion that there is a problem with the way the towngreen is used at the moment. The richest and smartest man in town, who also happens to be the governor of Connecticut, proclaims that the problem lies in the fact that most farmers are illiterate and thus cannot be expected to make wise decisions. He proposes that he take care of the towngreen and be allowed to charge a fee $f$ per cow that is sent there for grazing. He then donates the money to the town. Apart from this change, the number of cows is determined as in (a). Characterize the equilibrium if his proposal is accepted. Is there a chance the outcome will be efficient? If so, find the associated fee $f$ per cow, otherwise, briefly explain why the solution will still be inefficient.

Delila has wealth 1000. She is deciding what amount $x$ of her wealth she should put in a risky asset that yields a negative interest of $-10$ percent with probability 0.2 and a positive interest of $i$ percent with probability 0.8. Let Delila’s vNM utility function be $U(b) = \ln(b)$ where $b$ is her wealth level.

(a) (5 points) Set up Delila’s expected utility maximization problem.
(b) (10 points) Calculate Delila’s optimal investment decision $x$ as a function of $i$.

Alan and Betty live on a deserted island. The only consumption goods on this island are coconuts $C$ and pineapples $P$. Alan owns 100 coconuts and no pineapples and Betty owns 100 pineapples and no coconuts. Alan’s utility function is given by $U_A = P_A + 50\ln(C_A)$ where $P_A$ and $C_A$ are Alan’s consumption quantities. Betty’s utility function is $U_B = P_B + C_B$ where $P_B$ and $C_B$ are Betty’s consumption quantities.

Good Luck and Happy Holidays!
(a) (5 points) Draw an Edgeworth box for this economy (put pineapples on the horizontal axis) and sketch the utility indifference curves for both consumers.

(b) (5 points) Identify the core for this economy both analytically and in the graph.

(c) (12 points) Normalizing the price of pineapples to 1 and denoting the coconut price by \( p \), calculate \( A \)'s and \( B \)'s Marshallian demands for both goods as a function of \( p \) and then find all Walrasian equilibria for this economy.

**QUESTION 4 (20 POINTS)**

Consider a small open economy with two industries, \( X \) and \( Y \). The production functions are \( x = \min(K_X, L_X) \) and \( y = \min(2K_Y, L_Y) \). The country has factor endowments of \( \bar{K} \) and \( \bar{L} \). It faces world market prices of \( p^W_X = 3 \) and \( p^W_Y = 2 \).

(a) (15 points) Find the equilibrium production quantities and the equilibrium factor prices, assuming that the country produces both goods.

(b) (10 points) Sketch the Lerner-Pearce diagram for this economy (don’t forget to label).

**QUESTION 5 (23 POINTS)**

Anne (A) and Burt (B) both have the same income \( m \) at their disposal. With this income, they can buy a private good \( C \) and contribute to a charity. Call \( C_i \) person \( i \)'s \((i = A, B)\) private good consumption and \( G_i \) person \( i \)'s donation to the charity. The utility function of person \( i \) is given by \( U_i = C_i + \alpha_i \ln(G_A + G_B) \), where \( 0 < \alpha_A < \alpha_B < 1 \) and \( \alpha_A + \alpha_B = 1 \).

(a) (10 points) Find the Nash equilibrium charity contributions for Anne and Burt.

(b) (5 points) Find the Pareto efficient level \( G^* \) of total charity contributions.

(c) (8 points) Suppose the government introduces a per capita tax of \( G^*/2 \) which both Anne and Burt have to pay and gives the collected money to charity. Will the outcome be efficient? Do both Anne and Burt prefer the taxation equilibrium over the Nash equilibrium under a)?