ECONOMICS 6201 (FALL 2008)
FINAL EXAMINATION
100 POINTS TOTAL

QUESTION 1 (20 POINTS)
(a) (10 points) Cows are bought until the profit per cow is zero. The equilibrium number of cows is
\[ c = \frac{10000p^2}{a^2}. \]
The efficient solution can be found by setting the marginal total profit of the village equal to 0. This gives
\[ c = \frac{2500p^2}{a^2} \]
The profit per cow is then \( (a) \).
(b) (10 points) The average profit is now
\[ p \frac{100}{\sqrt{c}} - a - f. \]
Setting it equal to zero and solving for \( c \) gives
\[ c = \frac{10000p^2}{(a + f)^2} \]
This \( c \) is equal to the efficient number of cows if \( f = a \).

QUESTION 2 (15 POINTS)
(a) (5 points) Maximize
\[ 0.8 \ln[(1 + \frac{i}{100})x + 1000 - x] + 0.2 \ln[0.9x + 1000 - x]. \]
(b) (10 points) Taking the derivative with respect to \( x \), setting it equal to zero and solving yields
\[ x = \frac{8000i - 20000}{i} \]
which is the valid solution for \( i \in [\frac{20}{\pi}, \frac{40}{\pi}] \). If \( i > 20/7 \), \( x = 1000 \). If \( i < 20/8 \), \( x = 0 \).

QUESTION 3 (22 POINTS)
(a) (5 points) The Edgeworth box is a square with side length 100. \( B \)'s utility indifference curves are straight lines with slope -1. \( A \)'s indifference curves are downward-sloping and have the same slope for constant \( C \). They are tangent to \( A \)'s indifference curves at \( C_A = 50 \).
(b) (5 points) The core is a horizontal line at \( C_A = 50 \) and \( P_A \in [50\ln(2), 50] \).
(c) (12 points) Alan’s demands are \( C_A = \frac{50}{p} \) and \( P_A = 100p - 50 \) for \( p \geq 0.5 \). If \( p < 0.5 \), it is \( C_A = 100 \) and \( P_A = 0 \). Betty’s demands are \( C_B = \frac{100}{p} \) and \( P_B = 0 \) if \( p < 1 \), \( C_B = 0 \) and \( P_B = 100 \) if \( p > 1 \), and \( C_B + P_B = 100 \) if \( p = 1 \). Markets clear when \( p = 1 \). The Walrasian equilibrium quantities are then \( C_A = C_B = P_A = P_B = 50 \). \( p < 0.5 \) can’t be an equilibrium price because nobody would demand pineapples. \( 0.5 \leq p < 1 \) can’t be an equilibrium price.

Good Luck and Happy Holidays!
because pineapple demand would be smaller than supply. \( p > 1 \) can’t be an equilibrium price, either, because market clearing for the coconut market would suggest \( p = 0.5 \), a contradiction.

**QUESTION 4 (20 POINTS)**

(a) (15 points) Zero profit condition in the \( X \) industry:

\[
 w + r = 3,
\]

zero profit condition in the \( Y \) industry:

\[
 w + \frac{1}{2} r = 2.
\]

The equilibrium values are \( r = 2 \) and \( w = 1 \).

The factor market clearing conditions are

\[
 x + y = \bar{L},
\]

\[
 x + \frac{1}{2} y = \bar{K}.
\]

The equilibrium production quantities are \( y = 2(\bar{L} - \bar{K}) \) and \( x = 2\bar{K} - \bar{L} \). For this solution to be valid, we must impose \( \bar{L} \in [\bar{K}, 2\bar{K}] \).

(b) The Lerner-Pearce diagram shows the following lines: An isocost line starting at \((L, K) = (0, 0.5)\) and ending at \((L, K) = (1, 0)\), a unit revenue curve for the \( X \) industry which is a right angle with kink point \((1/3, 1/3)\), a unit revenue curve for the \( Y \) industry which is a right angle with kinkpoint \((1/2, 1/4)\) and a parallelogram which goes through the kinkpoints and up to the endowment point.

**QUESTION 5 (23 POINTS)**

(a) (10 points) Maximizing either person’s utility function by choice of \( G_i \), holding the other person’s contribution \( G_j \) fixed, leads to \( i \)’s reaction function \( G_i = \alpha_i - G_j \). Since \( \alpha_B > \alpha_A \), The Nash equilibrium in contributions occurs at \( G_B = \alpha_B \) and \( G_A = 0 \).

(b) (5 points) Maximizing the joint welfare of \( A \) and \( B \) gives \( G^* = 1 \).

(c) (8 points) First note that nobody will want to give additional money to charity if the government already gives \( G^* = 1 \). Further notice that from the text, we can infer that \( \alpha_B > 0.5 \) and \( \alpha_A < 0.5 \). Comparing utility levels for Nash and tax provision leads to the following inequalities:

\[
m - \alpha_B + \alpha_B \ln(\alpha_B) < m - 0.5,
\]

i.e. \( B \) is better off under government contribution and taxation, whereas \( m + \alpha_A \ln(\alpha_B) > m - 0.5 \)

i.e. \( A \) is better off in the private provision Nash equilibrium (since \( \alpha_A \ln(1-\alpha_A) \in (-0.34657, 0) \)).