Optimal Fiscal Policy and the Banking Sector

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Abstract

What should the government’s fiscal policy be when banks hold significant amounts of public debt and the government can default on its debt obligations? This question is addressed using a dynamic stochastic general equilibrium model where banks face constraints on their leverage ratios and adjust lending to satisfy regulatory requirements. In response to negative productivity shocks, the government subsidizes the banking sector by increasing bond repayments. This helps to sustain private sector lending. When government consumption exogenously increases, however, the government optimally taxes banks and partially defaults on its debt. Debt issuance is procyclical to ensure equilibrium in the deposit market. With an opening of the economy, the government uses less aggressive tax and default policies.

JEL Classification: E32; E62; F41; H21; H63

Key Words: Business Fluctuations; Debt; Fiscal Policy; Government Bonds; Ramsey Equilibrium; Optimal Taxation

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1 Introduction

Recent historical episodes suggest that governments are willing to expend large amounts of resources in order to support their banking sectors during recessions. For example, the Irish government spent roughly $83 billion, or 40 percent of Ireland’s 2009 GDP, bailing out six Irish banks during the recent financial crisis.\textsuperscript{1} Also, even as bond spreads between Germany and Spain were reaching euro era records in 2012, Spain’s prime minister pledged to not allow any Spanish bank fail and spent $24 billion, or close to two percent of Spain’s GDP, bailing out Bankia, a major Spanish bank.\textsuperscript{2}

In addition, there have been many examples of governments imposing capital controls during crises in an attempt to protect their banking sectors. According to IMF (2002) Ecuador, Russia, Ukraine, and Pakistan all imposed either capital controls or deposit freezes during their sovereign debt crises. In order to explain these policy actions, in this paper I present a model in which subsidies to the banking sector and taxes on foreign bank accounts are an optimal response to negative productivity shocks.

This paper examines optimal government tax, bond repayment, and debt issuance policies in a dynamic stochastic general equilibrium model with a banking sector. In the model the banking sector is subject to a constraint on its leverage ratio and so in response to a shock that reduces banks’ equity, banks optimally cut back on lending. Also, banks hold large amounts of government debt and so when the government partially defaults on its bonds, bank equity and private sector lending both decline. In Section 6.1 of the paper I demonstrate that, because of this, government defaults and bank taxes have harmful effects on the real economy.

In Section 6.2 I show that in this environment, the government optimally subsidizes


the banking sector in response to negative productivity shocks. However, the government partially defaults on bonds held by banks and taxes the banking sector when revenue needs exogenously increase (perhaps due to war). I also find that the government optimally imposes large taxes on workers’ savings in response to both negative productivity shocks and government consumption increases. These taxes have a negative income effect on workers which leads workers to reduce their consumption and increase their labor supply. Finally, government debt issuance is procyclical in the model. Given the large countercyclical taxes on workers, procyclical debt creation helps the government to balance its budget. Also, procyclical government debt issuance helps to ensure that there is an optimal amount of investment in the private economy.

In Section 7 of the paper I modify the model so that domestic banks must compete with foreign banks for deposits. In this environment the government optimally taxes foreign bank accounts in response to negative productivity shocks and the volatility of government defaults and domestic bank subsidies/taxes declines. However, when government consumption exogenously increases, foreign-deposit tax rates decline. This reduces the incentive for workers to increase their net foreign borrowing in response to the government shock.

I solve the Ramsey problem presented in this paper using the primal approach. I assume that the government has access to a commitment technology which allows it to commit to a set of policies in the initial period (as in Chari, Christiano, and Kehoe (1994)). I solve the model both in Dynare and via the discrete state-space method.

A key feature of the model is that government debt default and bank taxation lead to a decline in output through their impact on domestic bank lending. There is evidence that this mechanism is significant in reality. According to IMF (2002), when the Russian government defaulted on its bonds in 1998, over 30 percent of domestic bank assets were
restructured and the majority of large Russian banks failed. This led the cost of capital to increase dramatically and real credit to decrease by 12 percent. Gennaioli, Martin, and Rossi (2010) find that government default has a statistically significant and negative impact on credit to the private sector. Also, Borensztein and Panizza (2009) find that sovereign default is associated with a decline in output growth and an increase in the likelihood of a banking crisis.

In the model, default and bank taxation reduce private sector lending specifically through the negative impact that these policies have on banks’ capital adequacy ratios, or the ratio of banks’ equity to assets. Economist (2012) describes how European banks’ difficulty in meeting capital adequacy requirements has increased the severity of the recession in Europe:

“Banks are required to meet EU capital-adequacy targets by June. Raising fresh money is proving tricky...and so banks are rationing capital by selling assets. They are reluctant to make new loans...Industrial production in Poland, Hungary and the Czech Republic has held up surprisingly well so far, says Gillian Edgeworth, of UniCredit. But capital flows – including bank loans from the euro zone – are drying up.”

1.1 Related Literature

Similarly to this paper, Sosa (2012) and Gennaioli, Martin, and Rossi (2010) build models where sovereign default is economically costly because it negatively impacts the banking system and this in turn leads to a decline in private sector credit. These pa-

pers also analyze the circumstances under which a government would optimally decide to default on its debt. While both papers provide effective models for explaining how sovereign default can lead to a reduction in private sector credit and both papers are able to match certain aspects of the data, this paper offers an improvement in a number of areas. Specifically, the model in this paper includes capital and an endogenous level of bank equity.\(^4\) Also, I focus on the optimal size of government default over the business cycle and in response to both productivity shocks and government consumption shocks. In addition, I allow for the possibility that the government can subsidize, as well as default on, the banking sector and I allow the government to have access to many different tax instruments, all of which are distortionary. Finally, I examine optimal government behavior when the economy is open and depositors can move their deposits abroad if the domestic banking sector is providing poor returns.

This paper’s results differ significantly from the results of Sosa (2012) and Gennaioli, Martin, and Rossi (2010). Both find that the government optimally defaults when productivity is low whereas I find that the government optimally subsidizes the banking sector and accelerates bond repayments in response to negative productivity shocks. In my model, accelerating bond repayments improves banks’ equity position and this prevents banks from becoming severely leverage-ratio constrained for an extended period of time. This in turn prevents banks from significantly reducing their loans to the private sector and it mitigates the effects of a negative shock. So, this paper can help explain why governments often expend vast resources bailing out banks even when their fiscal position is weak.

The model structure is similar to Meh and Moran (2010) who build a model where,\(^5\)

\(^4\)Both Sosa (2012) and Gennaioli, Martin, and Rossi (2010) include an endogenous decision to hold government bonds which in turn impacts bank equity. However both models rely on exogenous endowments of banker wealth.
due to financial frictions, the quantity of bank equity determines how much banks are able to borrow from depositors and lend. This amount of lending in turn determines how much capital is purchased and how much output is produced. Meh and Moran (2010) use their model to analyze how banking sectors can amplify productivity and monetary policy shocks and explain how exogenous shocks to bank equity have negative economic consequences. In contrast, I focus exclusively on optimal government tax, debt, and bond repayment policies in a model where government debt is held by the banking sector. Also, Meh and Moran (2010) derive an endogenous equity constraint whereas I rely on an exogenous one. However, Meh and Moran (2010) assume that banks are risk neutral and an exogenous percentage of banks exit the economy each period. I assume that banks are risk averse and infinitely lived. This allows for a complex decision by bankers in terms of whether to distribute dividends or retain earnings and build up equity.

Following Atkinson and Stiglitz (1980), Chari and Kehoe (1999) and many others, I solve the model using the primal approach. Lockwood (2010) and others also employ the primal approach to discuss optimal taxation of the financial sector. Specifically, Lockwood (2010) analyzes optimal taxation of banks in the case where banks supply payment services to agents and in the case where banks act as financial intermediaries between households and firms (i.e. banks monitor firms on behalf of households). However, to the best of my knowledge this is the first paper to analyze optimal taxation of banks in a stochastic general equilibrium setting where banks are required to maintain adequate leverage ratios.

In this paper I present a real-goods economy. This enables me to focus exclusively on the optimal fiscal policy response to shocks. Because of this, the model is well

\[ \text{\footnote{Lockwood (2010) presents nonstochastic models.}} \]
suited for studying countries that are in currency unions like the euro. Dellas, Diba, and Loisel (2010) study optimal fiscal and monetary policy in an economy with a banking sector subject to financial frictions. Similarly to this paper, the authors find that when the banking sector experiences an exogenous increase in loan defaults, the government should optimally provide fiscal transfers. However, Dellas, Diba, and Loisel (2010) differ from this paper in a number of important respects. First, the authors present a monetary economy and assume there are price rigidities in place. Also, unlike this paper, the authors do not include a capital stock or government debt in their model and all taxes are lump sum rather than distortionary.6 Finally, financial frictions arise because there is a cost associated with altering bank dividends from their steady-state level (banks prefer to provide a smooth stream of dividends). I assume that financial frictions arise because banks are subject to a constraint on their equity-to-asset ratio.

The rest of the paper proceeds as follows. In Section 2 I describe the basic model. In Section 3 I present two possible scenarios for government policy. In the first, the government’s bond repayment decision is completely exogenous and stochastic. Sometimes the government pays creditors slightly more than what they are owed and sometimes it pays them slightly less. The purpose of this section is to show that government default is economically costly because it leads to a decline in private sector lending and output. I then present a scenario where the government faces a standard Ramsey problem: government consumption is exogenous and stochastic, but tax and bond repayment policies are set optimally. I use this scenario to derive the key model results. Section 4 describes the solution procedure and Section 5 describes the calibration procedure. Section 6 explains the results. Section 7 presents the slightly modified small open economy version of the model. In this section, I explore how the results change when workers have the

6Government bonds are implicitly included in their model in the sense that the central bank buys and sells these bonds in order to conduct monetary policy.
option of depositing their savings abroad. Section 8 provides a robustness check for the results and Section 9 concludes. Appendices A, B, and C provide proofs to propositions stated in the paper. Finally, Appendix D describes how to solve the model using the discrete state-space method and presents the results from using this solution procedure.

2 Model

Following Chari, Christiano, and Kehoe (1995) and others I adopt the following notation. \( s_t \) represents the realization of an exogenous event in period \( t \) and \( s^t \) represents the series of events prior to and including \( s_t \) (i.e., \( s^t = (s_0, ..., s_t) \)). For a given variable, \( x \), \( x(s^t) \) represents the value of \( x \) as a function of history \( s^t \). In the baseline (non-discrete state-space) model I assume there are an infinite number of possible realizations for \( s_t \) in each period.

2.1 Workers

Workers in the model supply labor, consume, and save. Workers seek to maximize the following:

\[
\sum_{t,s^t} \beta^t \pi(s^t) U^W(c(s^t), l(s^t)),
\]

where \( c \) is consumption of private goods, \( l \) is the quantity of labor supplied, \( \beta \) is the discount rate, \( \pi(s^t) \) is the probability of history \( s^t \) occurring and

\[
U^W(c(s^t), l(s^t)) = \frac{c(s^t)^{1-\sigma}}{1-\sigma} + \xi [1 - l(s^t)].
\]

Workers earn income by supplying labor and earning wage rate \( w \) and by earning interest on their savings, \( a \). They have one unit of time to divide between leisure and work. Each
period labor earnings are taxed at rate $\tau^l$. All money that workers save is deposited into a savings account where it earns a gross interest rate $R$. This interest income is taxed at rate $\tau^a$. Workers’ budget constraint is the following:

$$[1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})a(s^{t-1}) = c(s^t) + a(s^t).$$  \hspace{1cm} (3)$$

The intertemporal and intratemporal conditions are the following:

$$c(s^t)^{-\sigma} = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t)[1 - \tau^a(s^{t+1})]R(s^t)c(s^{t+1})^{-\sigma},$$  \hspace{1cm} (4)$$

$$[1 - \tau^l(s^t)]w(s^t)c(s^t)^{-\sigma} = \xi.$$  \hspace{1cm} (5)

2.2 Banks

Banks have liabilities, assets, and equity. Bank liabilities are the deposits that they accept from workers. Bank assets consist of loans to producers in the private sector and loans to the government. Each period banks lend firms capital, $k$, which firms then use for production. Loans to the government occur through the purchase of government bonds $b$. Bank equity, $n$, is equal to $k + b - a$ and will be described further below. The objective of bankers is to maximize the following:

$$\sum_{t,s^t} \beta^t \pi(s^t)U^B(m(s^t)), \hspace{1cm} (6)$$

where $m$ is banker consumption and

$$U^B(m(s^t)) = \frac{m(s^t)^{1-\sigma}}{1-\sigma}. \hspace{1cm} (7)$$

The representative bankers’ budget constraint is the following:
\[ m(s^t) + [1 + \gamma + \tau_e(s^t)]n(s^t) = R^k(s^t)k(s^{t-1})[1 - \tau^k(s^t)] + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - R(s^{t-1})a(s^{t-1}). \] 

(8)

In Equation (8) the right-hand side is the gross return on bank equity in period \( t \). \( R^k \) is the gross return on loans to producers, \( \tau^k \) is a tax on banks’ loan revenue, and \( \tau^e \) is a tax on banks’ equity. \( d \) is the default rate on government bonds and \( R^b \) is the gross return on government bonds that investors would receive if there was no default.

In Equation (8), \( \gamma \) represents a cost on bankers that is proportional to their equity position. This cost gives banks an added incentive to pay dividends instead of reinvesting their return on equity (i.e. allocating resources to \( m \) instead of \( n \)). This cost is motivated by the idea that as bank equity increases, so do principal-agent problems between bank owners and managers. For example, as equity increases managers may have more incentive and ability to devote resources to personal benefits and pet projects instead of to profit-maximizing investments. Many economists believe that principal-agent costs such as these can explain why firms (not just banks) pay dividends even though the effective capital-gains tax rate is lower than the effective dividend tax rate.\(^7\) \( \gamma \) is very important for solving the model.\(^8\)

I assume that banks are subject to an equity-ratio constraint that looks as follows:

\[ \mu n(s^t) \geq k(s^t), \] 

(9)

where \( \mu \) is a parameter that determines how much equity banks must hold in order to satisfy regulators. Government debt, \( b \), is not included as an asset in this equity-ratio

\(^7\)See Gruber (2010) for a discussion of this issue.

\(^8\)In the Ramsey problem described below, if \( \gamma \) did not exist then additional assumptions would be necessary in order to solve for \( n \).
constraint since regulators assign a zero risk weighting to government debt. Also, setting up the equity-ratio constraint like this allows me to solve Ramsey problem described below.  

Bankers maximize lifetime utility by lending capital to the point where:

$$
\beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) U_m^B(s^{t+1}) R^k(s^{t+1}) [1 - \tau^k(s^{t+1})] = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) U_m^B(s^{t+1}) R(s^t) + \chi(s^t),
$$

(10)

where $\chi$ is the Lagrange multiplier on the equity-ratio constraint. In Equation (10) the left-hand side is the expected marginal benefit of private sector lending and the right-hand side is the expected marginal cost. Bankers maximize lifetime utility by purchasing government bonds to the point where:

$$
\sum_{s^{t+1}} \pi(s^{t+1} | s^t) U_m^B(s^{t+1}) [1 - d(s^{t+1})] R^b(s^t) = \sum_{s^{t+1}} \pi(s^{t+1} | s^t) U_m^B(s^{t+1}) R(s^t).
$$

(11)

Finally, bankers maximize lifetime utility by purchasing $n$, i.e., reinvesting their return on equity, to the point where:

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9 Including government debt in the equity-ratio constraint would not significantly affect the results. Government default impacts the real economy through its impact on bankers’ equity-ratio constraint. Suppose that the government used all of its revenue from a default to pay down government debt and the equity-ratio constraint looked as follows: $\mu n(s^t) \geq k(s^t) + b(s^t)$. In this case, assuming dividends $m$ are held constant, the default would reduce $n$ and $b$ by equal amounts. However, as discussed in the calibration section, $\mu$ equals 12.5. So, the default still makes it much more difficult for banks to satisfy their equity ratio constraint.

---
\[ U^{B}_{m}(s^{t})[1 + \gamma + \tau^{e}(s^{t})] = \beta \sum_{s^{t+1}} \pi(s^{t+1} \mid s^{t})U^{B}_{m}(s^{t+1})R(s^{t}) + \mu \chi(s^{t}). \]  \hspace{1cm} (12)

Equation (12) is the Euler equation for bankers.

Typically in theoretical models with sovereign default, the government has two options when it comes to bond repayment: it can pay creditors back everything that it owes them or it can pay creditors nothing.\(^{10}\) This simplifying assumption makes sense when analyzing Argentina’s 2001 debt restructuring where the government imposed a 65 percent default on the majority of its debt.\(^{11}\) However, sovereign defaults are typically far less painful for creditors. Often times governments fail to honor their obligations by missing an interest payment or making an interest payment late. Also, governments often make agreements with creditors to reduce the interest rate or increase the maturity on public debt. In 2010, for example, the Jamaican government made an agreement of this nature with its creditors.\(^{12}\)

Due in part to these considerations, in the model I allow the government to partially default on its debt. Specifically, in Section 3.2 I assume that the government can set \(d\) to any value that it chooses. The government can even impose negative defaults whereby \(d\) is less than one.\(^{13}\) In this model environment, it makes sense to view default as one of

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\(^{10}\)See Arellano (2008) or Hatchondo, Martinez, and Sapriza (2009) for example.


\(^{13}\)In practice, the government could accomplish a negative default through deflation. Although money is not explicitly presented in this model, Chari and Kehoe (1999) show that in a stochastic economy with money, the government optimally uses monetary policy in order to adjust the real repayment of government borrowing.
the government’s many tax instruments where the default variable, $d$, is the tax rate on government bonds.

This modeling approach to government default is the same as in Chari and Kehoe (1999) who allow the government to have access to state contingent debt. I assume that $R^b(s^t)$ is determined in the period that bonds are issued, period $t$, and that $d(s^{t+1})$ is a function of the state of the economy in period $t + 1$. There is an indeterminacy in the model between $R^b(s^t)$ and $d(s^{t+1})$, and the return on government bonds could be presented as a single variable. I separate the return on government bonds into two variables for illustrative purposes. For example, if $R^b(s^t)$ is set to equal $R(s^t)$ in every period, then $d(s^{t+1})$ provides a clear measure of what bankers receive in each state relative to their required risk-free return.\footnote{This can be seen from the representative banker’s first order condition with respect to government bonds, Equation 11.}

### 2.3 Firms

Firms produce output, $Y$, using labor and capital according to the following production function:

\[
Y(s^t) = z(s^t)k(s^{t-1})^\alpha l(s^{t-1})^{1-\alpha},
\]  
(13)

where $z$ is total factor productivity (TFP) and it has a mean of one. TFP evolves according to the following equation:

\[
\log(z(s^t)) = \rho \log(z(s^{t-1})) + e^z(s_t),
\]  
(14)

where $e^z(s_t)$ is an exogenous and i.i.d. random shock with a mean equal to zero. Firms’ profit is equal to $Y(s^t) + (1 - \delta)k(s^{t-1}) - w(s^t)l(s^t) - R^c(s^t)k(s^{t-1})$ where $\delta$ is the
depreciation rate of capital. The markets for labor and capital are competitive and so input prices are equal to their marginal products. Specifically,

$$w(s^t) = (1 - \alpha)z(s^t)k(s^{t-1})^{\alpha}l(s^t)^{-\alpha}$$ (15)

and

$$R^k(s^t) = \alpha z(s^t)k(s^{t-1})^{\alpha-1}l(s^t)^{1-\alpha} + (1 - \delta).$$ (16)

### 2.4 Government

The government earns income from taxing workers’ labor earnings and deposit-interest income and from taxing bankers’ equity and private loan returns. The government also raises money by issuing bonds. All revenue is used for government consumption and for paying interest and principal on previously issued bonds. Finally, the government can reduce its need to raise revenue by defaulting on previously issued bonds. The government’s budget constraint is the following:

$$\tau^l(s^t)w(s^t)l(s^t) + \tau^a(s^t)R(s^{t-1})a(s^{t-1}) + \tau^k(s^t)R^k(s^t)k(s^{t-1}) + \tau^e(s^t)n(s^t) + b(s^t) = g(s^t) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}),$$ (17)

where $g(s^t)$ is government consumption.

In Section 3, I present two scenarios. In the first scenario, the government’s tax and default policies are exogenous. The purpose of this scenario is simply to demonstrate that in the model an unexpected government default leads to a ‘financial crisis’ and is associated with a contraction in bank lending and output. In the second scenario, tax and default policies are set optimally. Given that default is costly, this scenario demonstrates
how an optimizing government would alter debt, bond repayment, and tax policies in response to economic shocks.

2.5 Equilibrium

*Proposition 1*: Given the budget constraints of the representative worker, representative banker, and the government, the aggregate resource constraint is the following:

\[
Y(s^t) + (1 - \delta)k(s^{t-1}) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \gamma n(s^t). \tag{18}
\]

Proof: See Appendix A.

A government policy is a sequence of \(\tau_l(s^t), \tau_a(s^t), \tau_k(s^t), d(s^t), \tau_e(s^t), \) and \(R^b(s^t)\) for every \(s^t\). An allocation is a sequence of \(l(s^t), a(s^t), k(s^t), c(s^t), m(s^t), n(s^t), b(s^t)\) and \(g(s^t)\) for every \(s^t\). And a price system is a sequence of \(w(s^t), R(s^t),\) and \(R^k(s^t)\) for every \(s^t\).\(^{15}\)

A competitive equilibrium is defined as an allocation, government policy, price system, and initial values \(k_{-1}, a_{-1}, n_{-1}, R_{-1}, R^b_{-1},\) and \(b_{-1}\) that meet the following criteria: the allocation maximizes workers’ lifetime utility, (1), subject to (3), and the allocation maximizes bankers’ lifetime utility, (6), subject to (8) and (9); wages are given by (15) and the return on capital is given by (16); and the government satisfies its budget constraint, (17), in every period.\(^{16}\)

\(^{15}\)These definitions are similar to the ones in Chari, Christiano, and Kehoe (1994), Chari, Christiano, and Kehoe (1995), and Chari and Kehoe (1999).

3 Government Policy

3.1 Exogenous Government Policy

In this section I assume that government policy is exogenous and history independent. This allows me to clearly demonstrate that the model matches a key stylized fact: government default leads to a significant decline in bank lending and output.\textsuperscript{17} In the next section I derive optimal government policies given the costs of default.

The primary focus of this section is on the impact of government default on the model economy. So, for simplicity I assume that $\tau^a$, $\tau^e$, and $\tau^k$ are zero in all periods. $\tau^l$ is constant and set so that in the steady state, when $d$ equals zero and $z$ equals one, the government’s budget is balanced. Also I assume that the government adopts a simple decision rule for how to respond to a deficit or surplus:

$$\tilde{g} - g(s^t) = X[\tilde{g} + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - \tau^l w(s^t)l(s^t) - \tilde{b}]$$  \hspace{1cm} (19)

and

$$b(s^t) - \tilde{b} = (1 - X)[\tilde{g} + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - \tau^l w(s^t)l(s^t) - \tilde{b}].$$  \hspace{1cm} (20)

These equations show that in response to an increase in interest expenses or a decline in tax revenue, a share $X$ of the adjustment needed to balance the budget occurs through an decrease in spending and a share $(1 - X)$ of the adjustment occurs through an increase in debt.

For simplicity, I make $d$, the variable of interest, an i.i.d. random variable with a mean of zero. This implies that sometimes the government partially defaults and

\textsuperscript{17}Evidence on this stylized fact is discussed in the introduction.
sometimes the government repays banks more than expected. The latter case can be thought of as a subsidy to the banking sector.

I set $X$ to 0.1. When $X$ is large, shocks to $d$ are associated with large shocks to government consumption. Because of this, the negative impact that default has on the banking sector is offset to some extent by the expansionary impact that default has on government consumption. The purpose of this section is to demonstrate how default can lead to a reduction in economic activity through its impact on the financial sector and so, for ease of exposition, I set $X$ to a low number. However, when $X$ is too low $b$ does not converge to its steady-state value in response to a $d$ shock. Results for this model setup are described in Section 6.1.

### 3.2 Optimal Government Policy

In this section I assume that the government’s goal is to maximize the following social welfare function:

$$\sum_{t,s^t} \beta^t \pi(s^t)[U^W(c(s^t), l(s^t)) + \theta U^B(m(s^t))],$$

(21)

where $U^W(c(s^t), l(s^t))$ is given by Equation (2) and $U^B(m(s^t))$ is given by Equation (7). $\theta$ is an exogenous parameter and it describes the relative weight of bankers in the government’s social welfare function. One would expect $\theta$ to be large if, for instance, there are many citizens who work as bankers or if banks are major political contributors to the government.

In the last section, tax and default rates were completely exogenous while government consumption and debt levels reacted mechanically to changes in government revenue. Now however, only government consumption is exogenous while tax rates, default rates, and debt levels are set optimally. When the model is solved in Dynare, I assume
government consumption evolves according to:

\[
\log(g(s^t)) = (1 - \rho^g) \log(\bar{g}) + \rho^g \log(g(s^{t-1})) + e^g(s^t).
\]  

(22)

When the model is solved using the discrete-state method, I assume that government consumption follows a two-state Markov process.

I solve this Ramsey optimal taxation problem using the primal approach. I assume the government has access to a commitment technology where in the initial period it can commit to a policy for the rest of time. Once the government announces its policy, workers and bankers adopt allocation rules. These allocation rules determine allocations based on the announced government policy (this is the same as in Chari, Christiano, and Kehoe (1994), Christiano, and Kehoe (1995) and Chari and Kehoe (1999)).

Following Chari and Kehoe (1999), a Ramsey equilibrium is defined as a policy, allocation, price system, and initial values \(k_{-1}, a_{-1}, n_{-1}, R_{-1}, R_{b-1}, \) and \(b_{-1}, \) such that the government’s policy maximizes the social welfare function, Equation (21), subject to the aggregate resource constraint, Equation (18); and the requirements of a competitive equilibrium are satisfied.

**Proposition 2:** (i) The allocation implied by the Ramsey equilibrium maximizes (21), subject to the aggregate resource constraint in each period, Equation (18), the equity-ratio constraint in each period, Equation (9), and the following two implementability constraints:

\[
\sum_{t, s^t} \beta^t \pi(s^t)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)] = U^W_c(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1},
\]  

(23)
\[
\sum_{t,s^t} \beta^t \pi(s^t) U^B_m(s^t) m(s^t) = U^B_m(s_0)[[1 - \tau^k(s_0)] R^k(s_0) k_{-1} + [1 - d(s_0)] R^b_{-1} b_{-1} - R_{-1} a_{-1}].
\]

(ii) Given the fact that an allocation satisfies the above-mentioned constraints, it is possible to construct a government policy and a price system such that the allocation, government policy, and price system satisfy the definition of a competitive equilibrium.\(^{18}\)

Proof: See Appendix B.

Let \(\Lambda(s^t)\) be the Lagrange multiplier on the resource constraint for history \(s^t\), \(\chi^G(s^t)\) be the Lagrange multiplier on the equity-ratio constraint, \(\Phi\) be the the Lagrange multiplier on Equation (23), and \(\Gamma\) be the the Lagrange multiplier on Equation (24). Given Proposition 2, the government’s maximization problem can be written as: \(^{19}\)

\[
\max \sum_{t,s^t} \beta^t \pi(s^t) W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma)
+ \Lambda(s^t)[Y(s^t) + (1 - \delta) k(s^{t-1}) - c(s^t) - k(s^t) - m(s^t) - g(s^t) - \gamma n(s^t)]
+ \chi^G(s^t)[\mu n(s^t) - k(s^t)]
- \Phi U^W_c(s_0)[1 - \tau^a(s_0)] R_{-1} a_{-1}
- \Gamma U^B_m(s_0)[[1 - \tau^k(s_0)] R^k(s_0) k_{-1} + R^b_{-1} b_{-1} - R_{-1} a_{-1}]
\]

where

\(^{18}\)This proposition is similar to the ones presented in Chari, Christiano, and Kehoe (1994), Chari, Christiano, and Kehoe (1995) and Chari and Kehoe (1999).

\(^{19}\)Again, this is similar to Chari, Christiano, and Kehoe (1994), Chari, Christiano, and Kehoe (1995) and Chari and Kehoe (1999).
Results for this scenario are described in Section 6.2.

4 Solution Procedure

4.1 Solution Procedure for the Exogenous Policy Model

To solve this model I apply a second-order Taylor approximation around the non-stochastic steady state and solve using the standard perturbation method (in Dynare). However, this method is unable to solve models with occasionally binding constraints and so I replace the inequality constraint, Equation (9), with a penalty function similarly to Kim, Kim, and Kollman (2010). Specifically I change the representative banker’s utility function to look as follows:

\[ U^B(m(s^t), n(s^t), k(s^t)) = \frac{m(s^t)^{1-\sigma}}{1-\sigma} + \phi \log(\mu n(s^t) - k(s^t)) \] (27)

4.2 Solution Procedure for the Optimal Policy Model

Given the nature of the problem, the decision rules in period \( t = 0 \) are different than in period \( t > 0 \). Specifically, there is an incentive for the government to heavily tax deposits and impose a large default on its bonds in period zero. The reason for this is because in the initial period deposits and government debt are inelastically supplied, so taxation/default on these assets acts as a non-distortionary tax (see Chari and Kehoe (1999)). Due to this, economists usually place some sort of restriction on time-zero
policies. Following Benigno and Woodford (2006), I put restrictions on the right-hand sides of (23) and (24) rather than on initial tax and default rates per se. The restrictions are as follows:

\[ H^W = U^W_e(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1}, \]  

(28)

and

\[ H^B = U^B_m(s_0)[1 - \tau^k(s_0)]R^k(s_0)k_{-1} + R^b_{-1}[1 - d(s_0)]b_{-1} - R_{-1}a_{-1}, \]  

(29)

where \( H^W \) and \( H^B \) are parameters. I set \( H^W \) and \( H^B \) to the levels that would prevail when all variables are at their steady-state values in a non-stochastic version of the economy with \( z \) equal to one, and \( \bar{d}, \bar{a}, \bar{e}, \bar{k}, \bar{l}, \bar{b} \) and \( \bar{g} \) equal to their calibrated steady-state values as described below. I solve this model using the standard perturbation method as well as the discrete state-space method. More details about how the model can be solved using the latter method are discussed in Appendix D.

5 Calibration

This section focuses on the calibration procedure that is employed when the model is solved in Dynare. Appendix D provides details specific to the calibration of the discrete-state space version of the model. The only parameters that differ across solution methods are those relating to the stochastic processes of \( g \) and \( z \).

Table 1 lists the values for the calibrated parameters. \( \xi \) is calibrated so that in the “exogenous policy model” the steady-state value of labor, \( \bar{l} \), equals 0.19. According to U.S. Bureau of Labor Statistics (BLS) data from 1970 to 2011, on average people over
age 16 spend 19 percent of their total available time working.\(^{20}\) I set $\sigma$ to 1.5 which is within the standard range of estimated values as discussed by Mehra and Prescott (1985). Values for $\beta$ and $\delta$ come from Chari, Christiano, and Kehoe (1995). These values are calibrated to match the moments of annual U.S. data.

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>Capital Exponent</td>
<td>$\alpha$</td>
<td>0.36</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
<td>0.08</td>
</tr>
<tr>
<td>Utility from Leisure Parameter</td>
<td>$\xi$</td>
<td>9.89</td>
</tr>
<tr>
<td>Relative Importance of Bankers</td>
<td>$\theta$</td>
<td>0.0675</td>
</tr>
<tr>
<td>Coefficient of Relative Risk Aversion</td>
<td>$\sigma$</td>
<td>1.5</td>
</tr>
<tr>
<td>Capital Requirement Parameter</td>
<td>$\mu$</td>
<td>12.5</td>
</tr>
<tr>
<td>Technology Autocorrelation Parameter</td>
<td>$\rho^z$</td>
<td>0.72</td>
</tr>
<tr>
<td>Government Autocorrelation Parameter</td>
<td>$\rho^g$</td>
<td>0.85</td>
</tr>
<tr>
<td>Standard Deviation of Productivity Shocks</td>
<td>$\sigma^z$</td>
<td>0.012</td>
</tr>
<tr>
<td>Standard Deviation of Government Shocks</td>
<td>$\sigma^g$</td>
<td>0.018</td>
</tr>
<tr>
<td>Principal-Agent Cost Parameter</td>
<td>$\gamma$</td>
<td>0.048</td>
</tr>
<tr>
<td>Government Consumption Adjustment Share (Exogenous Policy Model)</td>
<td>$X$</td>
<td>0.1</td>
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<tr>
<td>Government Debt*</td>
<td>$\bar{b}/\bar{Y}$</td>
<td>46%</td>
</tr>
<tr>
<td>Government Consumption*</td>
<td>$\bar{g}/\bar{Y}$</td>
<td>22%</td>
</tr>
<tr>
<td>Deposit Tax Rate*</td>
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<tr>
<td>Bank-Equity Tax Rate*</td>
<td>$\bar{\tau}^e$</td>
<td>0%</td>
</tr>
<tr>
<td>Capital Tax Rate*</td>
<td>$\bar{\tau}^k$</td>
<td>0%</td>
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<tr>
<td>Labor Tax Rate*</td>
<td>$\bar{\tau}^l$</td>
<td>36.6%</td>
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<tr>
<td>Default Rate*</td>
<td>$\bar{d}$</td>
<td>0%</td>
</tr>
<tr>
<td>Foreign Deposit Adjustment Parameter</td>
<td>$\psi$</td>
<td>0.27</td>
</tr>
</tbody>
</table>

* Refers to steady-state values.

To calibrate $\gamma$, I use results from a study of agency costs in small businesses by Ang,\(^{20}\) I assume that people can work at most 16 hours per day 365 days per year.
Cole, and Lin (2000). The authors find that in firms where the primary owner owns 100 percent of the firm and also serves as the manager, the operating expense to sales ratio is 0.464. However in firms where no owner or family owns more than 50 percent of the firm and the firm is managed by an outsider, the operating expense to sales ratio is 0.553. This suggests that agency costs make up 8.9 percent of sales for firms in the latter scenario. The authors’ data also indicates that the average sales to assets ratio is 0.0465 which. Assuming for simplicity that the sales to assets ratio is constant across firms with different management/ownership structures, this suggests that agency costs make up 0.004138 percent of firms’ assets.\textsuperscript{21} Since 1988 the average assets-to-equity ratio at commercial banks has been 11.49. This suggests that agency costs make up roughly 0.048 percent of banks’ equity. In the model agency costs ($\gamma_n$) divided by equity ($n$) equals $\gamma$ and so I set $\gamma$ to 0.048 in the model.

To calibrate $\rho^z$ and $\sigma^z$, I use annual U.S. capital stock, output, and labor data from 1970 to 2011 from the Bureau of Economic Analysis (BEA) and the BLS.\textsuperscript{22} Specifically, I assume that output evolves according to:

$$Y_t = \exp(\rho t) z_t k_{t-1}^{\alpha} l_t^{1-\alpha},$$

where $\alpha$ is equal to 0.36. This is the value used in Kydland and Prescott (1982) based on estimates of the share of total U.S. income that accrues to capital owners. $l_t$ is equal to the average percentage of total available time that people over age 16 devote to work in year $t$. It is measured as the employment-population ratio multiplied by average annual

\textsuperscript{21}The authors actually find that firms that are managed by their owners and where the primary owner owns 100 percent of the firm have higher sales to assets ratios. However, for simplicity I assume that agency costs only impact the expense side of firms’ net income and so I ignore this result.

\textsuperscript{22}The reason for using this time period is because data on “average annual hours worked per employed person in the United States” is only available from 1970 to 2011.
labor hours per worker divided by total available hours per person. I then estimate the following equation using least squares:

\[ \Delta \log(Y_t) - \alpha \Delta \log(k_t) - (1 - \alpha) \Delta \log(l_t) = \hat{\varrho} + \epsilon_t \]  

(31)

and set \( \Delta \log(z_t) \) equal to \( \epsilon_t \). I use this to form a series for \( z_t \) and run the following regression:

\[ \log(z_t) = \hat{\phi}_0 + \hat{\phi}_1 \log(z_{t-1}) + \epsilon_t^z. \]  

(32)

I set the standard deviation of the productivity shock in the model, \( \sigma^z \), equal to the standard deviation of \( \epsilon^z \). Finally, I set \( \rho^z \) in the model equal to \( \hat{\phi}_1 \) in Equation (32).

I use annual U.S. data over the period 1970 to 2011 from the BEA to calibrate the steady-state value and stochastic process for government consumption, \( g \). Specifically, I use BEA data on “government consumption expenditures and gross investment.” This includes government expenditure and investment at the federal, state, and local levels. The steady-state value of \( g \) is calibrated so that, in the steady-state, government consumption divided by output equals the average value of this ratio in the data over the sample period. I calibrated \( \rho^g \) and \( \sigma^g \) by estimating the equation:

\[ \log(g_t) = \hat{\vartheta}_0 + \hat{\vartheta}_1 t + \hat{\vartheta}_2 \log(g_{t-1}) + \epsilon_t^g. \]  

(33)

Then I set \( \rho^g \) equal to \( \hat{\vartheta}_2 \) and set \( \sigma^g \) equal to the standard deviation of \( \epsilon^g \). To calibrate the steady-state level of government debt I use data from the IMF World Economic Outlook database on U.S. general government net debt over the period 1980 to 2011.\(^{23}\)

Recall that \( \mu \) is a parameter that dictates how much equity banks must hold relative to private sector loans and \( \theta \) is a parameter that dictates how much weight the govern-

\(^{23}\)This is as far back as the data goes.
ment puts on bankers’ utility. In the U.S. banks must hold eight cents in Tier 2 capital for every dollar they hold in risk-weighted assets and so $\mu$ is set to 12.5. In the baseline “optimal policy” model, $\theta$ is set to equal the steady-state ratio of banker consumption plus bank equity divided by worker consumption plus worker deposits from the “exogenous policy model.” As discussed in Section 8, the business-cycle characteristics of the allocations are not sensitive to the value of $\theta$.

In the model with exogenous government default policies, $\phi$, the penalty-function parameter used to approximate the occasionally binding equity-ratio constraint, is set so that in the steady-state bankers choose to hold 50 percent more equity than the minimum required amount (i.e. the equity to assets ratio is 0.12 instead of the legally required 0.08).\(^{24}\)

The procedure used to calculate time-zero policies (described in Section 4.2) allows for some freedom in terms of setting steady-state policies. For simplicity, I assume that in the steady state all tax revenue is raised from taxing labor and so $\bar{d} = \bar{\tau}^a = \bar{\tau}^e = \bar{\tau}^k = 0$. This implies that in the steady state, $\bar{R}^b = \bar{R}$. The steady-state tax rate on labor, $\bar{\tau}^l$, is then equal to:

$$\bar{\tau}^l = \frac{\bar{g} + (\bar{R} - 1)\bar{b}}{\bar{w}l}.$$  \hspace{1cm} (34)

Finally, in the small open economy model presented below, the adjustment cost parameter, $\psi$, is set to 0.27. This is because, when $\psi$ equals 0.27, policies are “exogenous,” and there are only productivity shocks as calibrated above, the model’s standard devia-

\(^{24}\)The penalty function, model structure and solution method are such that the range of possible values for the steady-state equity to risk-weighted assets ratio is restricted. That being said, Das and Sy (2012) study a sample of over 700 banks from over 30 countries and find that the average total capital to risk-weighted assets ratio was 0.1447 in 2006 and 0.1496 in 2010. This is relatively close to the steady-state value of 0.12.
tion of net exports divided by output is 1.73 percent. This is very close to the standard deviation of net exports divided by output in annual U.S. data from 1970 to 2011 (according to BEA data).

6 Results

6.1 Results for Exogenous Policy

Figure 1 shows the impulse responses associated with a government default shock. In the figure, the government has paid creditors (banks) only 99 percent of what it owes them. Default has an immediate and severe impact on investment. This is because in response to the default, banks reduce lending in order to rehabilitate their equity-to-asset ratios. This in turn leads to a decline in the capital stock and output. However, the decline in the capital stock leads to an increase in the marginal product of capital which leads to large returns for bankers on their remaining loans. These positive returns gradually improve bankers’ net worth which allows for a gradual increase in lending and output.

I solve the model using a second-order Taylor approximation, and so amplifying the size of the default has nonlinear impacts on the results. For example, in an economy with no productivity shocks increasing the standard deviation of \( d \), the default rate, from zero to one percent causes the standard deviation of output to increase by 0.013 percentage points. However, increasing the standard deviation of \( d \) from one to two percent causes the standard deviation of output to increase by an additional 0.02 percentage points.\(^{25}\)

Increasing the standard deviation of \( d \) increases the standard deviation of consump-

\(^{25}\)With the aforementioned penalty function in place, the model is unable to handle calibrations where the standard deviations of \( d \) is equal or larger than three percent.
tion for bankers and workers. For example, when the standard deviation of \( d \) increases from zero to two percent the standard deviation of banker consumption increases by 0.01 percentage points and the standard deviation of worker consumption increases by 0.02 percentage points. Due to the concave utility function of both workers and bankers, increases in volatility have a negative effect on utility.\(^{26}\)

### 6.2 Results for Optimal Policy

Before discussing the main results, it is important to note that with the utility function from Equation (2) in place, optimally the tax rate on labor is completely constant. To see this note that the intratemporal condition for the government is as follows:

\[
(1 - \alpha) z(s^t)k(s^{t-1})^\alpha l(s^t)^{-\alpha}c(s^t)^{-\sigma}[1 + \Phi(1 - \sigma)] = \xi(1 + \Phi).
\]

(35)

After noting that \( w \) is equal to the marginal product of labor (see Equation (15)), Equation (5) can be written as:

\[
(1 - \alpha) z(s^t)k(s^{t-1})^\alpha l(s^t)^{-\alpha}c(s^t)^{-\sigma} = \frac{\xi}{[1 - \tau^t(s^t)]}.
\]

(36)

Comparing (35) and (36) it is clear that the optimal policy is for the government to keep tax labor rates constant and equal to \( \Phi \sigma/(1 + \Phi) \). This is very similar to Chari and Kehoe (1995) who find that the optimal standard deviation of labor tax rates is extremely

\(^{26}\)However, the increase in the standard deviation of \( d \) also leads to an increase in the risk premium on government bonds. In the exogenous-policy scenario, this reduces government consumption (since government consumption is endogenous in this scenario). Because government consumption does not yield any utility in the model, the decline in government consumption has a positive effect on banker and worker utility. This makes the net welfare effects of increasing the standard deviation of \( d \) theoretically ambiguous in the exogenous-policy scenario.
Figure 1: Exogenous Policy Model Response to a One Percent Government Default

Notes: the default rate is presented in terms of its absolute deviation from zero and all other variables are presented in terms of percentage deviations from their steady-state values.
small.\footnote{In simulations where the productivity shock is calibrated to U.S. data, Chari and Kehoe (1995) show that the standard deviation of labor tax rates is only 0.1 percent over the business cycle.}

As discussed in Appendix B, there is an indeterminacy in the model and there are some variables that are not uniquely pinned down. However, the government’s optimal debt level, and the optimal tax rate on equity are uniquely determined. Also the model pins down the after-tax return on deposits in each state and the after-tax net income for bankers in each state. In other words,

\[ [1 - \tau^a(s^t)] R(s^{t-1}) a(s^{t-1}) \] (37)

and

\[ R^k(s^t)[1 - \tau^k(s^t)] k(s^{t-1}) + R^b(s^{t-1})[1 - d(s^t)] b(s^{t-1}) - R(s^{t-1}) a(s^{t-1}) \] (38)

are determined in each state. In order to turn these variables into something that resembles a tax rate, I compare what workers and bankers actually receive to what they would have hypothetically received in an economy that lacks deposit taxation, capital taxation, equity taxation, and defaults. In this economy, according to Equation (4), workers would receive

\[ \tilde{R}(s^t) = \frac{c(s^t)^{-\sigma}}{\beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) c(s^{t+1})^{-\sigma}} \] (39)

in an economy without deposit taxation. Therefore, I define

\[ \tilde{\tau}^a(s^t) = \frac{\tilde{R}(s^{t-1}) - [1 - \tau^a(s^t)] R(s^{t-1})}{\tilde{R}(s^{t-1})} \] (40)

In the steady state \( \tilde{\tau}^a(s^t) \) equals zero. This is clear from worker’s first order condition with respect to deposits, Equation (4). Similarly, in an economy without deposit
taxes, capital taxes, equity taxes or defaults, bankers would earn a gross rate of return of:

\[
\tilde{R}_e(s^t) = \left[ R^b(s^t) - \tilde{R}(s^{t-1}) \right] k(s^{t-1}) + \tilde{R}(s^{t-1}) n(s^{t-1}) \left[ 1 + \gamma \right] n(s^{t-1}).
\] (41)

on their equity. However, because these taxes are in fact in place, the return on equity ends up being:

\[
R_e(s^t) = \left[ R^b(s^t)[1 - \tau^b(s^t)]k(s^{t-1}) + R^d(s^t)[1 - d(s^t)]b(s^{t-1}) - R(s^{t-1})a(s^{t-1}) \right] \left[ 1 + \gamma + \tau^e(s^{t-1}) \right] n(s^{t-1}).
\] (42)

Then I define:

\[
\tilde{d}(s^t) = \frac{R_e(s^t) - R_e(s^t)}{R_e(s^t)}.
\] (43)

As with \( \tilde{\tau}^a(s^t) \), in the steady state \( \tilde{d}(s^t) \) equals zero.

\( \tau^e(s^t) \) and \( \tilde{d}(s^t) \) can be used to define a measurement for the total effective tax burden on bankers’ equity. I accomplish this by dividing the total tax burden on bankers in a given period by the total income bankers would have received if there were no finance-related taxation.28

\[
\tau^R(s^t) = \frac{\tau^e(s^t)n(s^t) + \tilde{d}(s^t)[R^b(s^t) - \tilde{R}(s^{t-1})]k(s^{t-1}) + \tilde{R}(s^{t-1})n(s^{t-1})}{[R^b(s^t) - \tilde{R}(s^{t-1})]k(s^{t-1}) + \tilde{R}(s^{t-1})n(s^{t-1})}.
\] (44)

Table 2 shows the business-cycle statistics for the model. These statistics indicate that the government actively employs taxes and subsidies on both workers and bankers. Bank tax rates are procyclical but deposit tax rates are countercyclical and the government issues debt procyclically. Interestingly, the percentage standard deviation of the

28The idea to define taxes in this way was inspired by Chari and Kehoe (1999).
The government’s debt level is large, at nearly twice the percentage standard deviation of output.

Table 2: Simulated Business-Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviations</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>0.25</td>
<td>0.86</td>
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<tr>
<td>Investment</td>
<td>3.49</td>
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<td>Government Consumption</td>
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<td>Bank Equity</td>
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<td>Private Sector Loans</td>
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<tr>
<td>Bank Deposits</td>
<td>0.42</td>
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</tr>
<tr>
<td>Government Debt</td>
<td>1.96</td>
<td>0.50</td>
</tr>
<tr>
<td>Default Rate ((\bar{d}))</td>
<td>1.18</td>
<td>0.81</td>
</tr>
<tr>
<td>Bank Equity Tax Rate ((\tau^e))</td>
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<tr>
<td>Total Bank Tax Rate ((\tau^B))</td>
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</tr>
<tr>
<td>Deposit Tax Rate ((\tau^a))</td>
<td>0.49</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

Notes: all standard deviations are in percentage form except for the tax-rate standard deviations. All data is HP filtered with a penalty parameter of 6.25. These statistics are based on a 400 period simulation as in Chari, Christiano, and Kehoe (1995).

Figure 2 shows the impact of a one standard deviation negative productivity shock. The negative productivity shock reduces wages and, due to the substitution effect, this encourages workers to reduce their labor supply. To combat this, the government optimally increases deposit tax rates. This transmits a negative wealth effect to workers that incentivizes workers to reduce their consumption and increase their labor supply, all else equal. Overall however, the substitution effect from the shock overwhelms the government-imposed wealth effect and the labor supply declines on impact.

The government optimally subsidizes banks in response to a negative shock. The

\[^{29}\text{Dellas, Diba, and Loisel (2010) also find that imposing negative wealth effects is the optimal response to certain types of financial shocks.}\]
Figure 2: Endogenous Policy Model Response to a One Standard Deviation Negative Productivity Shock

Notes: all tax rates are presented in terms of absolute deviations from their steady-state values and all quantities are presented in terms of percentage deviations from their steady-state values.
government achieves these subsidies through a decrease in $\tilde{d}$ and $\tau^e$ in the period of the shock. These subsidies help banks to muddle through the recession without becoming too capital constrained. If the government did not do this, then the impact of the negative shock on banks’ balance sheets would serve to amplify the recession as in Meh and Moran (2010). Conversely, the government finds it optimal to increase $\tilde{d}$ and $\tau^e$ in response to positive productivity shocks.

Comparing the government and the representative banker’s first-order conditions can provide further insight into the reasons behind these policy responses. The government’s first-order condition with respect to bank equity is the following:

$$\gamma m(s^t)^{-\sigma}[\theta + \Gamma(1 - \sigma)] = \mu \chi^G(s^t),$$  \hspace{1cm} (45)

where $\chi^G$ is the Lagrange multiplier associated with the equity-ratio constraint in the government’s Ramsey problem. The representative banker’s first-order condition is the following:

$$m(s^t)^{-\sigma}[1 + \gamma + \tau^e(s^t)] = \beta \sum_{s^{t+1}} \pi(s^{t+1} \mid s^t)m(s^{t+1})^{-\sigma} R(s^t) + \mu \chi(s^t).$$  \hspace{1cm} (46)

As Equation (govnfoc) shows, the government prefers to set $n(s^t)$ exactly at the point where the equity-ratio constraint binds at all times. This minimizes principal-agent costs associated with having large amounts of bank equity in the economy and at the same time satisfies the equity-ratio constraint. However, holding tax rates constant, bankers may prefer to let bank equity build relative to private sector loans during times when interest rates are high and the cost of accepting deposits is large. Also, bankers have more incentive to accumulate equity (save) when expected future banker consumption is low relative to current banker consumption. These situations tend to occur during economic expansions. Alternatively, bankers have a stronger incentive to
become equity constrained during times when interest rates are low and expected future banker consumption is high relative to current consumption. These situations tend to occur during economic contractions. So, the government taxes banks during good times in order to discourage what it views as an unnecessary build up of equity. Also, the government subsidizes bank equity during bad times to ensure that equity-constrained banks do not curtail private sector lending too severely.30

As Figure 2 shows, the government only defaults and taxes deposits during the period of the shock and these policy instruments are not used in subsequent periods. This result is related to the following proposition:

**Proposition 3:** Optimally, the value of expected future revenue from \( \tilde{d} \) and \( \tilde{\tau}^{a} \) is zero in every period. More specifically,

\[
\sum_{s^{t+1}} q(s^{t+1}) \tilde{\tau}^{a}(s^{t+1}) \tilde{R}(s^{t}) a(s^{t}) = 0 \quad \forall t, \tag{47}
\]

and

\[
\sum_{s^{t+1}} q(s^{t+1}) \tilde{d}(s^{t+1}) \tilde{R}^{e}(s^{t+1}) n(s^{t}) = 0 \quad \forall t, \tag{48}
\]

where \( q(s^{t+1}) \) is the period \( t \) value of a state \( s^{t+1} \) good and \( q(s^{t+1}) = \beta \pi(s^{t+1} | s^{t}) c(s^{t+1})^{-\sigma} / c(s^{t})^{-\sigma} \) as in Chari, Christiano, and Kehoe (1994).

**Proof:** See Appendix C.

Proposition 3 shows that, optimally, the government should not distort the saving and investment decisions of workers and bankers. The model results are very similar

---

30 Another way that the government could potentially adjust equity levels is by altering regulatory requirements over the course of the business cycle (i.e. altering \( \mu \)). Berka and Zimmermann (2011), for example, analyze the impact of altering capital requirements during recessions. This policy is outside the scope of this paper. Also one can imagine that the exogenous regulatory requirement is set by depositors (not the government) in order to avoid potential losses.
to the results of Chari, Christiano, Kehoe (1994) who find that the government should optimally set the value of expected capital tax rates to zero but that asset taxation should be used as a “shock absorber.” This result in turn is a stochastic version of Chamley’s (1986) proof that capital tax rates should equal zero in the steady state.

Surprisingly, the government reduces its debt load in response to the negative shock. The reason is that during the downturn workers want to reduce their savings (to smooth consumption) at a sharper rate than the government wants to reduce investments in the private economy. To accommodate workers the government optimally reduces government bonds in the economy. In practice, one way the government may introduce and remove bonds from the economy is through central bank ‘quantitative easing’ programs. Also, the reduction in debt reflects the increase in revenue associated with the increase in deposit tax rates. The positive correlation between debt and output contrasts strongly with the seminal work of Barro (1979) who finds that the government should optimally reduce tax revenue during a downturn and it should finance this reduction by issuing debt.

Figure 3 shows the optimal government response to an exogenous one standard deviation increase in government consumption. Interestingly, the government finds it optimal to increase both $\tilde{d}$ and $\tilde{\tau}_a$ in response to the shock. These actions impose a negative wealth effect on workers and bankers. This leads workers and bankers to reduce their consumption and it leads workers to increase their labor supply. This is optimal in response to a government consumption increase (Dellas, Diba, and Loisel (2010)).

Rather than partly finance the consumption shock by issuing debt, the government finds it optimal to significantly reduce its debt level in response to the shock. Similarly to before, this response is optimal because it enables workers’ deposits to decline while at the same time allowing the capital stock to increase. The build up of capital helps the
Figure 3: Endogenous Policy Model Response to a One Standard Deviation Positive Government Consumption Shock

Notes: all tax rates are presented in terms of absolute deviations from their steady-state values and all quantities are presented in terms of percentage deviations from their steady-state values.
economy to accommodate the increased demands from the government. Also as before, the
government’s imposition of negative wealth effects on agents, via taxation, helps to produce a surplus.

As in Section 6.1, increasing the standard deviation of shocks has a nonlinear impact on both policies and allocations. For example, consider the case where the standard deviation of government consumption shocks and the standard deviation of productivity shocks both increase by 100 percent. This leads the standard deviation of output, the standard deviation of the default variable, $\tilde{d}(s^t)$, and the standard deviation of total bank taxes to all increase by 101 percent. The standard deviation of $\tilde{d}(s^t)$, for example, increases from 1.91 percent to 3.84 percent. Perhaps this result can partially explain why developing countries, which tend to be subjected to larger economic shocks, default on their debt more frequently than developed countries.

The indeterminacy described in Appendix B can also be resolved by assuming that the government lacks the ability to use capital taxation as a policy instrument (i.e. $\tau^k$ is always zero) and by setting $R^b$ equal to $R$ in every period.\textsuperscript{31} When the indeterminacy is resolved in this way, the results are mostly similar to before. Specifically, the undetermined variables, $d$, $\tau^a$, and $\tau^B$, respond in the same direction as before to shocks. $\tau^a$ increases in response to both negative productivity shocks and positive government consumption shocks. Also as before, $d$ and $\tau^B$ decrease in response to negative productivity shocks and increase in response to positive government consumption shocks. The HP-filtered correlations with output are very similar to before. The HP-filtered relative standard deviations of $\tau^a$ and $\tau^B$ are very similar to their previous values as well. However, the HP-filtered relative standard deviation of $d$ declines by 42 percent.

\textsuperscript{31}As noted in the Appendix, when this assumption is made, it is important to make sure that the Kuhn-Tucker condition, $\chi \geq 0$ holds. When this condition does not hold, capital tax rates should be adjusted. In the simulations described in this section, $\chi$ was greater than zero in every period.

37
Events from the recent European debt crisis suggest that even governments under severe financial distress are willing to spend large sums of money in order to ensure their banking sectors are adequately capitalized. These actions coincide with the results because in the model the government optimally subsidizes the banking sector in response to negative productivity shocks.

7 Small Open Economy Model

According to IMF (2002) during the sovereign restructuring episodes in Ecuador, Pakistan, Russia, and Ukraine people attempted to move their savings out of domestic financial institutions and into foreign bank accounts. The governments in these countries took active measures to prevent these attempts. This pattern raises interesting questions. Specifically, are capital controls an optimal response to sovereign default induced banking crises? Also, how does optimal government policy depend on the openness of the economy?

I attempt to answer these questions by altering the model so that workers have a choice about where to put their savings. Workers can invest their money in the domestic banking sector as before, or they can invest their money in banks abroad. From here on I refer to this specification as the ‘small open economy’ version of the model. In this section, I compare the results in the small open economy model to the results in the baseline (closed) model.

When workers invest abroad, they earn the risk-free world interest rate, $R^w$, which for simplicity is constant and equal to $1/\beta$. The representative worker’s budget constraint now looks as follows:
\[ [1 - \tau_l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})a^d(s^{t-1}) \\
+ [1 - \tau_f(s^{t-1})]R^w a^f(s^{t-1}) = c(s^t) + a^d(s^t) + a^f(s^t) + \psi \bar{a}^f(s^t)a^f(s^t). \] (49)

The term on the far right represents a cost associated with investing or borrowing from abroad and is needed to close the model. Specifically, \( \psi \) is a foreign-asset adjustment cost parameter, and \( \bar{a}^f \) represents average foreign assets across all workers in the economy. Importantly, workers do not consider how their decisions will affect \( \bar{a}^f \) when deciding how much foreign assets to hold since there are infinitely many workers who are all identical.\(^{32}\) In order to maintain the proper degrees of freedom in the Ramsey problem, I assume \( \tau_f(s^{t-1}) \) is set prior to the realization of period \( t \) shocks. Workers’ first order condition with respect to foreign deposits is the following:

\[ c(s^t)^{-\sigma}[1 + \psi \bar{a}^f(s^t)] = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t)[1 - \tau_f(s^t)]R^w c(s^{t+1})^{-\sigma}. \] (50)

Now the Ramsey problem is the following:\(^{33}\)

\[
\begin{align*}
\max_{t, s^t} & \sum_{t, s^t} \beta^t \pi(s^t)W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) \\
& + \Lambda(s^t)[Y(s^t) + (1 - \delta)k(s^{t-1}) + R^w a^f(s^{t-1}) - c(s^t) - k(s^t) - m(s^t) - g(s^t) \\
& - a^f(s^t) - \psi a^f(s^t)^2 - \gamma n(s^t)] \\
& + \chi^G(s^t)[\mu n(s^t) - k(s^t)] \\
& - \Phi U^W_m(s_0)[1 - \tau^a(s_0)]R_{a-1}^d + [1 - \tau^f_{s-1}]R^w a^f_{s-1}] \\
& - \Gamma U^B_m(s_0)[1 - \tau^k(s_0)]R^k(s_0)k_{s-1} + R^p_{s-1}[1 - d(s_0)]b_{s-1} - R_{a-1}^d, \\
\end{align*}
\] (51)

\(^{32}\)This is similar to the “big K, little k trick” described by Sargent and Ljungqvist (2004).

\(^{33}\)The proof of this follows along the same lines as the proof of Proposition 2.
where \( W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) \) is defined as:

\[
W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) = \left[ U^W(c(s^t), l(s^t)) + \theta U^B(m(s^t)) \right] \\
+ \Phi[U^W_C(s^t)c(s^t) + U^W_L(s^t)l(s^t)] + \Gamma U^B_m(s^t)m(s^t).
\]

(52)

The government considers aggregate foreign assets when making decisions and so \( \bar{a}^f \) is replaced with \( a^f \) when modeling the government’s problem.

### 7.1 Exogenous Policy Results for Small Open Economy Model

Figure 4 shows that government default has more devastating consequences in the open economy setting compared to the closed economy setting. In both the closed and open economy cases default leads to a decline in the demand for deposits and a reduction in the interest rate on deposits. In the open economy case this leads to an increase in deposits held overseas. In the small open economy model, default leads to a sharper decline in the quantity of domestic deposits supplied and, due to this, a sharper decline in domestic bank lending.

### 7.2 Optimal Policy Results for Small Open Economy Model

Table 3 and Figures 5 and 6 present results for when policy is set optimally and the economy is open. Tax-related variables are, for the most part, less volatile relative to output when the economy is open. In response to a negative productivity shock, total bank tax rates, \( \tau^B \), decline less if the economy is open than if it is closed. This is because when the economy is open, workers can earn interest abroad and protect against a decline in interest earnings. So, optimally there is a larger decline in domestic financial intermediation when the economy is open. This implies that when the economy is open the government should sustain the banking sector less in response to a negative
Figure 4: Small Open Economy Exogenous Policy Model Response to a One Percent Government Default

Notes: the default rate is presented in terms of its absolute deviation from zero and all other variables are presented in terms of percentage deviations from their steady-state values.
productivity shock. Also, when the economy is open, taxes have a more distortionary impact on behavior. As such, less taxation is needed in order for the government to achieve its desired impact on allocations.

Table 3: Simulated Business-Cycle Statistics

<table>
<thead>
<tr>
<th>Variable (x =)</th>
<th>Closed Standard Deviation</th>
<th>Open Standard Deviation</th>
<th>Closed Correlations</th>
<th>Open Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>0.25</td>
<td>0.24</td>
<td>0.86</td>
<td>0.88</td>
</tr>
<tr>
<td>Investment</td>
<td>3.49</td>
<td>4.62</td>
<td>0.98</td>
<td>0.91</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>0.85</td>
<td>0.82</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.28</td>
<td>0.32</td>
<td>0.65</td>
<td>0.84</td>
</tr>
<tr>
<td>Private Sector Loans</td>
<td>0.28</td>
<td>0.32</td>
<td>0.65</td>
<td>0.84</td>
</tr>
<tr>
<td>Domestic Bank Deposits</td>
<td>0.42</td>
<td>0.46</td>
<td>0.70</td>
<td>0.82</td>
</tr>
<tr>
<td>Foreign Bank Deposits</td>
<td>0.11</td>
<td></td>
<td>-0.91</td>
<td></td>
</tr>
<tr>
<td>Government Debt</td>
<td>1.96</td>
<td>1.93</td>
<td>0.50</td>
<td>0.53</td>
</tr>
<tr>
<td>Default Rate (d̅)</td>
<td>1.18</td>
<td>1.17</td>
<td>0.81</td>
<td>0.76</td>
</tr>
<tr>
<td>Bank Equity Tax Rate (τε)</td>
<td>0.18</td>
<td>0.13</td>
<td>0.61</td>
<td>0.09</td>
</tr>
<tr>
<td>Total Bank Tax Rate (τB)</td>
<td>1.30</td>
<td>1.21</td>
<td>0.82</td>
<td>0.74</td>
</tr>
<tr>
<td>Domestic Deposit Tax Rate (τa)</td>
<td>0.49</td>
<td>0.47</td>
<td>-0.51</td>
<td>-0.49</td>
</tr>
<tr>
<td>Foreign Deposit Tax Rate (τf)</td>
<td>0.03</td>
<td></td>
<td>-0.91</td>
<td></td>
</tr>
</tbody>
</table>

Notes: all standard deviations are in percentage form except for the foreign deposit and tax-rate standard deviations. All data is HP filtered with a penalty parameter of 6.25. These statistics are based on a 400 period simulation as in Chari, Christiano, and Kehoe (1995).

Regardless of whether the economy is open or closed, in response to a positive government consumption shock the government increases tax rates on the domestic banking sector. However, the government increases tax rates less when the economy is open compared to when it is closed. This is partly because when the economy is open, workers have the option of investing their savings abroad. Increasing bank tax rates too significantly would lead too much assets to exit the domestic financial system and it
Figure 5: Small Open Economy Endogenous Policy Model Response to a One Standard Deviation Negative Productivity Shock

Notes: all tax rates are presented in terms of absolute deviations from their steady-state values and all quantities are presented in terms of percentage deviations from their steady-state values.
would make it difficult for the capital stock to increase, as it should in response to the positive government shock. However, the government must balance this consideration with the need to impose a negative wealth effect on bankers.

Figure 6: Small Open Economy Endogenous Policy Model Response to a One Standard Deviation Positive Government Consumption Shock

Notes: all tax rates are presented in terms of absolute deviations from their steady-state values and all quantities are presented in terms of percentage deviations from their steady-state values.

The government raises tax rates on foreign deposits in response to a negative productivity shock and reduces tax rates on foreign deposits in response to a positive government consumption shock. To understand the intuition behind this result, note that the government’s first order condition with respect to foreign deposits is the following:
\[ c(s^t)^{-\sigma} \left[ 1 + \Phi(1 - \sigma) \right] \left[ 1 + 2\psi a^f(s^t) \right] = \beta \sum_{s^{t+1}} \pi(s^{t+1} \mid s^t) \left[ 1 + \Phi(1 - \sigma) \right] R^w c(s^{t+1})^{-\sigma}. \]  

(53)

Comparing Equations (50) and (53) and noting that in equilibrium \( \bar{a}^f(s^t) = a^f(s^t) \) since all workers are identical, one can see that tax rates on foreign deposits are equal to:

\[ \tau^f(s^t) = \frac{\psi a^f(s^t)}{1 + 2\psi a^f(s^t)}. \]  

(54)

Equation (54) shows that foreign deposit tax rates are perfectly correlated with foreign deposits.

The government employs foreign deposit taxation entirely to correct the externality associated with foreign investment. Workers do not consider the impact that their foreign investment has on average foreign deposit levels and so Pigovian taxation is optimal. However, if there were no externalities, i.e., \( \bar{a}^f = a^f \) for workers, then workers’ first order condition with respect to foreign deposits would be:

\[ c(s^t)^{-\sigma} \left[ 1 + 2\psi a^f(s^t) \right] = \beta \sum_{s^{t+1}} \pi(s^{t+1} \mid s^t) \left[ 1 - \tau^f(s^t) \right] R^w c(s^{t+1})^{-\sigma}. \]  

(55)

Comparing this to Equation 53, it is clear that foreign-deposit taxes would equal zero in all periods. This finding suggests that if there are no externalities associated with removing deposits from the domestic banking system, then capital controls and deposit freezes are not an optimal response to recessionary periods. The intuition behind this result is that when the economy is open there exists a saving technology for workers that is outside of the domestic banking system, and it is optimal for workers to utilize this technology to help insulate themselves from the effects of shocks.

The results provide important insights for understanding countries’ behavior during economic crises. Ecuador froze bank deposits during its sovereign-debt crisis, Ukraine
and Pakistan imposed capital controls during their sovereign restructuring episodes, and Russia froze deposits and imposed capital controls during its 1998 crisis (IMF (2002)). According to the model, these actions were efficient to the extent that they helped reduce adjustment costs in an environment where such costs were not fully internalized by agents. A bank run on domestic banks, which leads productive loans to be inefficiently liquidated (as in Diamond and Dybvig (1983)), can perhaps be thought of as an adjustment cost of this sort.

8 Robustness Test

In this section I examine the effects of increasing the relative weight that the government puts on bankers’ utility (see Equation (21)). Specifically, I change $\theta$ and period-zero policies in such a way that the steady-state value of $\bar{m}$ increases.\textsuperscript{34}

The business-cycle statistics for most allocations and policy variables are not sensitive to changes in the relative weight of bankers. When the steady-state value of $\bar{m}$ increases by roughly 720 percent for example, the (non-HP-filtered) percentage standard deviation of output, worker consumption, and banker consumption all stay the same. The standard deviation of defaults also remains unchanged. However, under this scenario the standard deviation of bank-equity taxation increases by 700 percent. This reflects the fact that the government must employ bank-equity taxes more aggressively in order to increase banker consumption while keeping bank equity levels roughly unchanged.\textsuperscript{35}

\textsuperscript{34}In practice, this entails first increasing $\bar{m}$ and then finding the values of $\theta$, $\tau^e$, and $\tau^a$ that would coincide with this value of $\bar{m}$ in a non-stochastic version of the model economy. As before, I assume that in the non-stochastic steady state, $d = \tau^k = 0$, and $\bar{b}$ and $\bar{g}$ are calibrated as in Section 5, and $z$ equals one.

\textsuperscript{35}Also, the business-cycle statistics for government debt, domestic deposits, and $\tilde{\tau}^a$, change very mod-
To understand why allocations are not sensitive to bankers’ relative importance, it is helpful to look at the government’s first-order condition with respect to banker consumption:

\[ [1 + \Phi(1 - \sigma)]c(s^t)^{-\sigma} = [\theta + \Gamma(1 - \sigma)]m(s^t)^{-\sigma}. \tag{56} \]

The log-linear approximation of this equation is \( \tilde{c}(s^t) = \tilde{m}(s^t) \) where \( \tilde{c}(s^t) \) and \( \tilde{m}(s^t) \) represent the percentage deviation of these variables from their steady-state values. Due to this, the correlation between banker consumption and worker consumption is one. Also, when the steady-state level of banker consumption increases, the steady-state level of worker consumption decreases but the aggregate share of total resources devoted to private sector (worker plus banker) consumption remains constant. So, when banker importance increases, the government increases banker consumption at the expense of worker consumption but the two variables are perfectly correlated and so this change has no impact on most allocations’ business-cycle statistics.\(^{36}\)

9 Conclusion

This paper presents a model where banks channel workers’ savings into private investments and government bonds. Banks are subject to an equity-ratio constraint and so a sovereign default or other negative shock to bank equity has harmful effects on private

\(^{36}\)It should be noted that when banker importance increases, there is a wealth effect on workers that leads them to increase their labor supply. This increases steady-state output but it does not influence business-cycle statistics for most of the allocations. The labor supply is not very sensitive to the importance of bankers. For example, when steady-state banker consumption increases by 3,400 percent, the steady-state labor supply increases by only 2.1 percent.
sector lending and output. In this context I solve for optimal tax, debt, and bond repayment policies using the primal approach. I assume that the government can commit to a set of optimal policies at the start of time.

Results indicate that optimally the government should actively adjust bond repayments and bank tax rates in response to shocks. In response to a negative productivity shock, the government optimally increases bond repayments to the banking sector. This reduces the severity of banks’ equity-ratio constraint and reduces the amount that banks cut back on lending in response to the shock.

Also, in response to a negative productivity shock, the government significantly increases taxes on workers’ deposits and reduces its debt. The tax on deposits has a negative income effect on workers and this encourages workers to reduce their consumption and increase their labor supply. The decline in government debt enables the supply of deposits by workers to meet the demand for deposits by banks while at the same time ensuring that a socially optimal amount is invested in the private economy. Also, given the tax increase on workers, reducing government debt helps to satisfy the government’s budget constraint.

In response to an exogenous increase in government consumption, the government partially defaults on its bonds and increases tax rates on workers’ deposits. This policy response efficiently provides for the increase in government consumption by imposing negative wealth effects on workers and bankers. These negative wealth effects lead to an increase in the labor supply and a decline in private sector consumption. Also, the government reduces its debt level in response to the shock. Optimally, workers’ deposits decline but private sector lending increases. Reducing the government’s debt level enables bankers to increase investment in the economy even as workers’ savings decline. Here again, the reduction in government debt helps guarantee that the government’s
budget constraint holds.

The relative standard deviations of tax rates are lower when the economy is open compared to when it is closed. A key reason for this is that taxes are more distortionary when savers have the option of investing their savings abroad. Also, in response to a negative productivity shock, the government taxes foreign deposits.

Overall, the results can help explain many examples of government behavior in practice. Most importantly, the results help explain why even fiscally strained governments often are willing to spend vast sums to bail out their banking sectors. In the model, such behavior helps to stave off a significant contraction in lending.

Future research should attempt to add optimal monetary policy into this model setting and look for ways to incorporate an endogenous equity-ratio constraint on the banking sector.\textsuperscript{37} Also, it would be productive to see how optimal policies change when the government lacks the ability to commit to a set of policies. In addition, it would be interesting to see how optimal policies differ when the government spends its money on valuable public goods instead of wasteful consumption and when some government bonds are held by foreigners in a model similar to the one presented. Adding these features to the model would help provide more accurate quantitative results with regard to how much and under what circumstances the government should tax the banking sector and alter bond repayments.

\textsuperscript{37}Dellas, Diba, and Loisel (2010) look at both optimal fiscal and optimal monetary policy but as described above their model structure is very different because the authors do not include capital, government bonds, or distortionary taxation in their model. Also, Meh and Moran (2010) derive endogenous equity constraints but the authors don’t study optimal policy.
A Proof of Proposition 1

The purpose of this appendix is to prove Proposition 1, that the aggregate resource
constraint is equal to:

\[ Y(s^t) + (1 - \delta)k(s^{t-1}) = c(s^t) + m(s^t) + k(s^t) + g(s^t) \]

\[ + \gamma n(s^t). \]  

(57)

First note that the aggregate resource constraint is the sum of the representative worker, the representative banker, and the government’s budget constraint. The representative worker’s budget constraint is the following:

\[ [1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})a(s^{t-1}) = c(s^t) + a(s^t). \]  

(58)

By the definition of bank equity:

\[ a(s^t) = k(s^t) + b(s^t) - n(s^t). \]  

(59)

So, the worker budget constraint can be written as:

\[ [1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})[k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})] = 
\]

\[ c(s^t) + k(s^t) + b(s^t) - n(s^t). \]  

(60)

The banker budget constraint is the following:

\[ R^k(s^t)[1 - \tau^k(s^t)]k(s^{t-1}) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) 
\]

\[ -R(s^{t-1})[k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})] = m(s^t) + [1 + \gamma + \tau^e(s^t)]n(s^t), \]  

(61)

and the government’s budget constraint can be written as:

\[ \tau^l(s^t)w(s^t)l(s^t) + \tau^a(s^t)R(s^{t-1})[k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})] 
\]

\[ +R^k(s^t)\tau^k(s^t)k(s^{t-1}) + \tau^e(s^t)n(s^t) + b(s^t) = g(s^t) + 
\]

\[ R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}). \]  

(62)
Summing the three budget constraints leads to the aggregate resource constraint:

\[ w(s^t)l(s^t) + R^k(s^t)k(s^t-1) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \gamma n(s^t). \] (63)

From firms’ profit maximization problem, the following condition holds:

\[ Y(s^t) + (1 - \delta)k(s^t-1) = w(s^t)l(s^t) + R^k(s^t)k(s^t-1). \] (64)

And so the aggregate resource constraint is:

\[ Y(s^t) + (1 - \delta)k(s^t-1) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \gamma n(s^t). \] (65)

**B Proof of Proposition 2**

Recall that Proposition 2 states the following: (i) the allocation implied by the Ramsey equilibrium maximizes (21), subject to the aggregate resource constraint in each period, Equation (18), the equity-ratio constraint in each period, Equation (9), and the following two implementability constraints (restated for convenience):

\[ \sum_{t,s'} \beta^{t'} \pi(s') [U_c^W(s')c(s') + U_l^W(s')l(s')] = U_c^W(s^0)R_{-1} [1 - \tau^c(s^0)]a_{-1} \] (66)

and

\[ \sum_{t,s'} \beta^{t'} \pi(s') U_m^B(s')m(s') = U_m^B(s^0)[R^k(s^0)[1 - \tau^k(s^0)]k_{-1} + R_{-1}^b[1 - d(s^0)]b_{-1} - R_{-1}a_{-1}]. \] (67)
(ii) Given the fact that an allocation satisfies the above-mentioned constraints, it is possible to construct a government policy and a price system such that the allocation, government policy, and price system satisfy the definition of a competitive equilibrium.

The proof follows along the same lines as Chari, Christiano, and Kehoe (1995). Recall that a Ramsey equilibrium maximizes the government’s social welfare function subject to two criteria: the aggregate resource constraint holds and the requirements of a competitive equilibrium are satisfied. So, to prove (i) one must show that satisfying these two criteria is equivalent to satisfying constraints (18), (9), (66), and (67). To show this, note that there are four requirements for a competitive equilibrium to be satisfied. First, the allocation has to solve the representative worker’s maximization problem. Second, the allocation has to solve the representative banker’s maximization problem. Third, wages and the return on capital must equal to the marginal products of labor and capital respectively. And fourth, the government’s budget constraint must be satisfied. Due to Walras’ Law this fourth requirement is equivalent to requiring the representative worker’s budget constraint, representative banker’s budget constraint, and aggregate resource constraint to all hold. Due to Weitzman (1973) and Ekeland and Sheinkman (1986), we know that solving the representative worker’s maximization problem and the representative banker’s maximization problems is equivalent to finding an allocation that satisfies certain conditions. The conditions are these agents’ first-order conditions, their constraints, and their transversality conditions.\(^{38}\) So, the constraints necessary and sufficient for the four requirements of a competitive equilibrium to be satisfied are the following:

\[
w(s^t) = (1 - \alpha) z(s^t) k(s^{t-1})^\alpha l(s^t)^{-\alpha},
\]

\(^{38}\)The idea to use these authors’ theorems comes from Chari, and Kehoe (1999).
\[ R^k(s^t) = \alpha z(s^t)k(s^t-1)^{\alpha-1}l(s^t)^{1-\alpha} + (1 - \delta), \]  
\[ (69) \]

\[ Y(s^t) + (1 - \delta)k(s^t-1) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \gamma n(s^t), \]  
\[ (70) \]

\[ \beta^t \pi(s^t) U_c^W(s^t) \leq \lambda(s^t) \]  
\[ (71) \]

with equality if \( c(s^t) > 0 \),

\[ \beta^t \pi(s^t) U_1^W(s^t) \leq -\lambda(s^t)[1 - \tau^t(s^t)]w(s^t) \]  
\[ (72) \]

with equality if \( l(s^t) > 0 \),

\[ [\lambda(s^t) - \sum_{s^t+1} \lambda(s^{t+1})[1 - \tau^a(s^{t+1})]R(s^t)]a(s^t) = 0, \]  
\[ (73) \]

\[ \lim_{t \to \infty} \sum_{s^t} \lambda(s^t)a(s^t) = 0, \]  
\[ (74) \]

\[ [1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})a(s^{t-1}) = c(s^t) + a(s^t), \]  
\[ (75) \]

\[ \beta^t \pi(s^t) U_m^B(s^t) \leq \omega(s^t) \]  
\[ (76) \]

with equality if \( m(s^t) > 0 \),

\[ \sum_{s^t+1} \omega(s^{t+1})[[1 - d(s^{t+1})]R^b(s^t) - R(s^t)] = 0, \]  
\[ (77) \]
\[
\sum_{s^{t+1}} \omega(s^{t+1})[R^k(s^{t+1})[1 - \tau^k(s^{t+1})] - R(s^t)] - \chi(s^t)]k(s^t) = 0,
\] (78)

\[
[\omega(s^t)[1 + \gamma + \tau^e(s^t)] - \sum_{s^{t+1}} \omega(s^{t+1})R(s^t) - \mu \chi(s^t)]n(s^t) = 0,
\] (79)

\[
\lim_{t \to \infty} \sum_{s^t} \lambda(s^t)b(s^t) = 0,
\] (80)

\[
\lim_{t \to \infty} \sum_{s^t} \lambda(s^t)k(s^t) = 0,
\] (81)

\[
\lim_{t \to \infty} \sum_{s^t} \lambda(s^t)n(s^t) = 0,
\] (82)

\[
R^k(s^t)[1 - \tau^k(s^t)]k(s^{t-1}) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1})
\]

\[
- R(s^{t-1})a(s^{t-1}) = m(s^t) + [1 + \gamma + \tau^e(s^t)]n(s^t)
\]

\[
\mu n(s^t) \geq k(s^t),
\] (84)

where \(\lambda(s^t)\) is the Lagrange multiplier associated with the representative worker’s budget constraint, Equation (75), and \(\omega(s^t)\) is the Lagrange multiplier associated with the representative banker’s budget constraint, Equation (83).

Equations (71) to (75) are from the representative worker’s problem. Equation (71) is the representative worker’s first-order condition with respect to consumption, (72) is the first-order condition with respect to labor, (73) is the first-order condition with respect to deposits, (74) is the transversality condition for deposits and (75) is the representative worker’s budget constraint. The next nine equations are from the representative bankers’ problem. Equations (76), (77), (78), (79) are the representative banker’s
first-order conditions with respect to consumption, government bonds, private loans, and equity respectively. Equations (80), (81), and (82) are the transversality conditions for government bonds, private loans, and equity respectively. (83) is the representative banker’s budget constraint. Finally, Equation (84) is the representative banker’s equity ratio constraint.

If an allocation satisfies (71) to (75), then it satisfies (66). Similarly to Chari and Kehoe (1999), this can be seen by multiplying (75) by \( \lambda(s^t) \) and summing over all \( t \) and \( s^t \). Then using (73) and (74), this can be written as:

\[
\sum_{t,s^t} \lambda(s^t)[c(s^t) - (1 - \tau^t(s^t))w(s^t)l(s^t)] = \lambda(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1}.
\] (85)

Then (71) and (72) imply:

\[
\sum_{t,s^t} \beta^t \pi(s^t)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)] = U^W_c(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1}.
\] (86)

Similarly, if an allocation satisfies (76) to (88), then it also satisfies (67) and (9). This can be seen by multiplying (88) by \( \omega(s^t) \) and summing over all \( t \) and \( s^t \). Using (78) to (79) this can be manipulated to obtain:

\[
\sum_{t,s^t} [\omega(s^t)m(s^t) + \chi(s^t)[\mu n(s^t) - k(s^t)]] = \\
\omega(s^0)U^R_m(s^0)[R^k(s^0)[1 - \tau^k(s^0)]k_{-1} + R^b_{-1}[1 - d(s^0)]b_{-1} - R_{-1}a_{-1}].
\] (87)

By the Kuhn-Tucker condition,

\[
\chi(s^t)[\mu n(s^t) - k(s^t)] = 0 \ \forall t, s^t.
\] (88)

Using (76) and (88), Equation (87) can be written as:
\[
\sum_{t,s} \beta^t \pi(s^t)U^B_m(s^t)\mu(s^t) = U^B_m(s^0)[R^k(s^0)[1-\tau^k(s^0)]k_{-1} + R^b_{-1}[1-d(s^0)]b_{-1} - R_{-1}a_{-1}] 
\]

(89)

Moving on to the second part of the proof: that a price system, policy, and allocation that represents a competitive equilibrium can be obtained from an allocation that satisfies (18), (9), (66), and (67). To see this, observe that it is possible to recover the competitive equilibrium value of government debt, \(b(s^t)\), in each period when the variables associated with the allocation satisfy (18), (9), (66), and (67). Following Chari and Kehoe (1999), this is clear from multiplying (75) by \(\lambda(s^t)\) and summing across all \(t\) and \(s^t\). From Equations (71) to (75) and the fact that \(a(s^t) = k(s^t) + b(s^t) - n(s^t)\) for a specific realization of a history, \(s^r\), this is equal to:

\[
U^W_c(s^r)[k(s^r) + b(s^r) - n(s^r)] = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \pi(s^t | s^r)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)](90)
\]

or

\[
U^W_c(s^r)[k(s^r) + b(s^r) - n(s^r)] = \sum_{s^{r+1}} \beta \pi(s^{r+1} | s^r)U^W_c(s^{r+1})c(s^{r+1}) + U^W_l(s^{r+1})l(s^{r+1}) + U^W_c(s^{r+1})[k(s^{r+1}) + b(s^{r+1}) - n(s^{r+1})].
\]

(91)

In the discrete-state space model, I solve for \(b(s^{r+1})\) similarly to Chari, Christiano, and Kehoe (1994). Specifically, I make use of the fact that in the government’s optimization problem \(k(s^{r+1})\) is the only endogenous state variable (this is not the case in the decentralized problem). Then Equation (91) can be written for every possible combination of \(k(s^r)\), \(g(s^r)\), and \(z(s^r)\). I solve the system of equations to find the government’s recursive decision for \(b(s^{r+1})\)
To recover $\tau^t$ I make use of the following condition:

$$\frac{U^W_t(s^t)}{U^W_c(s^t)} = [1 - \tau^t(s^t)](1 - \alpha)z(s^t)k(s^t-1)^\alpha l(s^t)^{-\alpha}. \quad (92)$$

Equation (92) is the result of combining Equations (15), (71) and (72). Next, to recover $\tau^e(s^t)$ I use Equations (78) to (79), (83), and (88) to arrive at:

$$\omega(s^t)[1 + \gamma + \tau^e(s^t)]n(s^t) = \sum_{s^t+1} \omega(s^{t+1})[m(s^{t+1}) + [1 + \gamma + \tau^e(s^{t+1})]n(s^{t+1})], \quad (93)$$

Similarly to how Equation (91) can be used to solve for $b(s^t)$, Equation (93) can be used to solve for $\tau^e(s^t)$.

Since the government maximizes its social welfare function subject to the banker’s equity-ratio constraint, Equation (84) will be satisfied in all circumstances. The government’s first order condition with respect to $n(s^t)$ is the following:

$$\gamma c(s^t)^{-\sigma}[1 + \Phi(1 - \sigma)] = \mu \chi^G(s^t), \quad (94)$$

where $\chi^G(s^t)$ is the government’s Lagrange multiplier for the equity-ratio constraint. Since the left side of Equation (94) is always positive, the right side must be too. So, in order for the government’s Kuhn-Tucker condition,

$$\chi^G(s^t)[\mu n(s^t) - k(s^t)] = 0 \quad \forall t, s^t, \quad (95)$$

to hold, it must be the case that

$$\mu n(s^t) = k(s^t) \quad \forall t, s^t. \quad (96)$$

39 This is similar to Chari, Christiano, and Kehoe (1994).
Since \( \mu n(s') \) always equals \( k(s') \), the representative banker’s Kuhn-Tucker condition, Equation (88), will always hold regardless of the value of \( \chi(s') \). This then implies that \( \chi(s') \) in the representative banker’s problem can take any value so long as the Kuhn-Tucker condition,

\[
\chi(s') \geq 0 \quad \forall t, s',
\]
holds.

Now it must be stated that there is an indeterminacy in the model: there are many possible price systems and policies that, when combined with an allocation that satisfies (9), (18), (66), and (67), represent a competitive equilibrium. To see this, first note that \( \hat{R}_b(s't-1)[1 - d(s')] \) can be replaced with a new variable called \( \hat{R}_b(s') \).\(^{40}\) Solving for the variables \( R(s'), \tau^e(s'), \) and \( \chi(s') \) requires finding the values of these variables at every grid point in the state space. If there are \( n^k \) possible values of \( k \), \( n^g \) possible values of \( g \), and \( n^z \) possible values of \( z \) in the state space, then solving for the three variables listed above would require a system of \( 3 \times n^k \times n^g \times n^z \) linear equations. The values of \( \hat{R}_b(s'+1) \) and \( \tau^k(s'+1) \) depend not only on the current state space but also on the outcome of shocks next period. So, to solve for these two variables requires a system of \( 2 \times n^k \times n^g \times n^z \) linear equations. Writing (78) to (79) and (83) for every possible combination of relevant state variables provides a system of \( 3 \times n^k \times n^g \times n^z + n^k \times n^g \times n^z \times n^z \) linear equations. So, \( R(s'), \tau^e(s'), \chi(s'), \hat{R}_b(s') \) and \( \tau^k(s') \) are under-identified by \( n^k \times n^g \times n^z \) equations.

There are many possible ways to resolve this indeterminacy. For example, one solution would be to set capital tax rates to zero in all periods except for when it would imply that the Kuhn-Tucker condition \( \chi(s') \geq 0 \) is not satisfied. In these cases, one

\(^{40}\)As mentioned earlier, this is how Chari, Christiano, and Kehoe (1994) label state-contingent government debt in their model.
could set $\chi(s^t)$ to zero and then also restrict capital tax rates so that they must be set one period in advance.

Assuming that the indeterminacy is resolved in this manner, Equations (78) and (79) can be used to solve for $R(s^t)$ and either $\chi(s^t)$ or $\tau^b(s^t)$ (remember that $\tau^c(s^t)$ was already solved for using the procedure described above). Then Equation (83) can be used to solve for $\tilde{R}^b(s^t)$. It is important to remember $\tau^c(s^t)$ was set, in part, by imposing that Equation (77) must be satisfied. This ensures that Equation (77) is satisfied when setting $\tilde{R}^b(s^t)$.

Given the nature of the model, $R^b(s^{t-1})$ and $d(s^t)$ can be set to any values so long as $\tilde{R}^b(s^t) = R^b(s^{t-1})[1 - d(s^t)]$. For example, I could set $R^b(s^{t-1})$ as the highest $\tilde{R}^b(s^t)$ across all states in period $t$. Then $d(s^t)$ would measure the discount relative to the highest feasible payout on government bonds. One can think of the highest feasible payout on government bonds as the bonds’ face value.

Finally, given $R(s^{t-1})$, Equation (75) determines $\tau^a(s^t)$ in each state. Similarly to $\tau^c(s^t)$ and Equation (77), $b(s^t)$ was set so as to guarantee that Equation (73) is satisfied in all states. Hence, I do not need to worry about Equation (73) being satisfied when setting $R(s^{t-1})$ or $\tau^a(s^t)$.

\section*{C \ Proof of Proposition 3}

Proposition 3 states that optimally:

$$\sum_{s^{t+1}} q(s^{t+1}) \tilde{\tau}^a(s^{t+1}) \tilde{R}(s^t) a(s^t) = 0 \quad \forall t, \quad (98)$$

and
\[ \sum_{s^{t+1}} q(s^{t+1}) d(s^{t+1}) \tilde{R}^e(s^{t+1}) n(s^t) = 0 \quad \forall t, \quad (99) \]

where \( q(s^{t+1}) = \beta \pi(s^{t+1} | s^t) c(s^{t+1})^{-\sigma} / c(s^t)^{-\sigma} \).

To see that Equation (98) holds, first note that it can be rewritten as:

\[ \sum_{s^{t+1}} q(s^{t+1}) R(s^t) - q(s^{t+1}) [1 - \tau^a(s^t)] R(s^t) = 0 \quad \forall s^t, t, \quad (100) \]

because of the definition of \( \tilde{\tau}^a \) and because \( \tilde{R}(s^t) \) and \( a(s^t) \) are determined in period \( t \).

This then can be rewritten as:

\[ \sum_{s^{t+1}} [1 - q(s^{t+1}) [1 - \tau^a(s^t)] R(s^t)] = 0 \quad \forall s^t, t, \quad (101) \]

by the definition of \( \tilde{R}(s^t) \). Finally, the representative worker’s Euler equation, Equation (4), makes it clear that Equation (101) holds.

Similarly, Equation (99) can be rewritten as:

\[ \sum_{s^{t+1}} q(s^{t+1}) \tilde{R}^e(s^{t+1}) - R^e(s^{t+1}) = 0 \quad \forall s^t, t. \quad (102) \]

Also,

\[ \tilde{R}^e(s^{t+1}) = \frac{[R^k(s^t) - \tilde{R}(s^{t-1})] \mu n(s^t) + \tilde{R}(s^{t-1}) n(s^t)}{1 + \gamma} \quad (103) \]

since the government always sets \( \mu n(s^t) = k(s^t) \). Then,

\[ \sum_{s^{t+1}} q(s^{t+1}) \tilde{R}^e(s^{t+1}) = \frac{\mu q(s^{t+1}) R^k(s^t) + 1 - \mu}{1 + \gamma}. \quad (104) \]

Combining the government’s first order conditions with respect to bank equity and capital it can be shown that
\[ q(s^{t+1})R^k(s^t) = 1 + \gamma / \mu, \quad (105) \]

and so

\[ \sum_{s^{t+1}} q(s^{t+1}) \hat{R}^e(s^{t+1}) = 1. \quad (106) \]

Also,

\[
\sum_{t,s^{t+1}} q(s^{t+1}) R^e(s^{t+1}) = \frac{\sum_{t,s^{t+1}} q(s^{t+1})[R^k(s^{t+1})[1 - \tau^k(s^{t+1})] - R(s^t)]\mu n(s^t) + R(s^t)n(s^t)]}{[1 + \gamma + \tau^e(s^t)]n(s^t)},
\]

\[ (107) \]

after imposing \( \mu n(s^t) = k(s^t) \), the representative banker’s first order condition with respect to government bonds, and the government’s first order condition with respect to banker consumption. Using the representative banker’s first order condition with respect to private sector loans this can be rewritten as:

\[
\sum_{s^{t+1}} q(s^{t+1}) R^e(s^{t+1}) = \frac{\chi(s^t)\mu/m(s^t) + \sum_{t,s^{t+1}} q(s^{t+1}) R(s^t)}{1 + \gamma + \tau^e(s^t)}.
\]

\[ (108) \]

Then, using the representative banker’s first order condition with respect to equity, this can be rewritten as:

\[
\sum_{t,s^{t+1}} q(s^{t+1}) R^e(s^{t+1}) = \frac{1 + \gamma + \tau^e(s^t) - \sum_{t,s^{t+1}} q(s^{t+1}) R(s^t) + \sum_{t,s^{t+1}} q(s^{t+1}) R(s^t)]}{1 + \gamma + \tau^e(s^t)},
\]

\[ (109) \]

which equals one. This, along with (106), then implies that (99) holds.
D The Discrete-State Space Model

The purpose of this Appendix is to present and discuss the solution procedure, calibration, and results for the closed-economy optimal policy model in the case where the discrete state-space method is used to solve the model. The discrete state-space method allows for a universal solution, not just for a solution within a small neighborhood around the steady state. Also, assuming that the state space is large enough, a discrete-state space based solution will provide a more accurate solution than even a second-order Taylor approximation around the non-stochastic steady state.\footnote{For a discussion on the strengths and weaknesses of discrete state-space methods, see Burnside (1999).} For these reasons, solving the model using the discrete-state space method provides an effective robustness check of the results.

D.1 Solution Procedure and Calibration for the Discrete-State Space Model

In the discrete-state space model, steady-state calculations as described in Section 4 are used to find $\Phi$ and $\Gamma$ as well as starting values for the capital stock.\footnote{In the Ramsey model, bank equity is a jumper variable and so it does not require an initial value.} I assume that there are four exogenous states: $(g_h, z_h)$, $(g_h, z_l)$, $(g_l, z_h)$, and $(g_l, z_l)$, where $g$ and $z$ are the values for the government and technology shock respectively, and $h$ stands for ‘high’ and $l$ stands for ‘low’. I set $g_h$ and $g_l$ so that the percentage standard deviation of government consumption in the model matches the standard deviation of the detrended logarithm of government consumption in the data (annual U.S. data over the period 1970 to 2011). $g_h$ is one percentage standard deviation above and $g_l$ is one percentage standard deviation below the non-stochastic steady-state value of government.
consumption. Similarly, \( z_h \) and \( z_l \) are set so that the percentage standard deviation of \( z \) in the model matches the standard deviation of the logarithm of \( z \) in the data (\( z \) is calculated as described in Section 5). I use values from Chari, Christiano, and Kehoe (1995) to calibrate the probability of transitioning from one state to another. The probability of \( g \) transitioning to a new state is assumed to be independent of the whether or not \( z \) transitions to a new state and vice versa. Table 4 shows the calibrated values for the relevant parameters. All of the other parameter values are the same as in the baseline model.

The size of the capital grid is 216 points, the size of the labor grid is 141 points and the size of the banker-consumption grid is 51 points.\(^{43}\) Because the government always forces the equity-ratio constraint to bind at equality, there is no need for a bank-equity grid. Also, it is important to note that banker consumption and worker labor are intratemporal decisions and for the government the only endogenous state variable in the model is capital. This implies that the model can be solved in two steps. First I solve for optimal labor input and optimal banker consumption at every possible combination of period \( t \) capital, period \( t + 1 \) capital, government consumption, and productivity level. Then, given that the social planner always makes the labor and banker-consumption choices optimally, I can iterate to convergence on the Bellman equation in order to find the optimal period \( t + 1 \) capital stock given the period \( t \) capital stock, government consumption, and productivity level. Solving the model in this manner reduces the memory demands on the computer. Using a computer with 2.0 gigabytes of RAM and an Intel Core 2 Duo processor (basically a standard personal laptop from 2008) it takes

\(^{43}\)The distance between grid points is the following: 0.012 units between points in the capital grid (the non-stochastic steady state is 1.12), 0.0035 units between points in the labor grid (the non-stochastic steady state is 0.19), and 0.00008 units between points in the banker-consumption grid (the non-stochastic steady state is 0.0029).
one hour and 58 minutes to solve the model in MATLAB.

Table 4: Calibration for Discrete-State Space Model

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP in high productivity state</td>
<td>$z_h$</td>
<td>1.017</td>
</tr>
<tr>
<td>TFP in low productivity state</td>
<td>$z_l$</td>
<td>0.983</td>
</tr>
<tr>
<td>High Government Consumption</td>
<td>$g_h$</td>
<td>0.082</td>
</tr>
<tr>
<td>Low Government Consumption</td>
<td>$g_l$</td>
<td>0.076</td>
</tr>
<tr>
<td>Probability of switching technology states</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>Probability of switching government states</td>
<td></td>
<td>0.05</td>
</tr>
</tbody>
</table>

D.2 Results for the Discrete-State Space Model

Table 5 presents the results for the discrete-state space version of the model. As the results show, government debt continues to be more volatile than output. As before, this is because the government actively injects and withdraws debt from the economy to make sure that the supply of deposits by workers matches the demand for deposits by banks while ensuring that an optimal level of private investment takes place. As in the baseline model, the results suggest that government debt decreases in response to negative productivity shocks and in response to positive government consumption shocks. However, now the correlation between output and government debt is close to zero and the relative volatility of HP-filtered debt has declined.

As before, in response to negative productivity shocks the government increases bond repayments to bankers. This ensures that bank equity does not decline excessively during recessions. However, now the relative standard deviation of the default variable, $\tilde{d}$, is significantly larger than before but equity tax rates, $\tau_e$, are negatively correlated
with output. Since \( \tilde{d} \) is more volatile than \( \tau^e \), the total bank tax rate, \( \tau^B \), still declines in response to negative productivity shocks. So, although the precise manner in which the government utilizes its policy tools has changed, overall, the government still subsidizes banks in response to negative productivity shocks.

Finally, as before, the government taxes deposits in response to negative productivity shocks. This allows the government to impose a negative wealth effect on workers. Overall, the results suggest that the key conclusions of the model do not depend on the solution method employed.
Table 5: Simulated Business-Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma(x)/\sigma(Y)$</th>
<th>$\rho(x,Y)$</th>
<th>$\rho(x,Z)$</th>
<th>$\rho(x,G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>0.34</td>
<td>0.18</td>
<td>0.44</td>
<td>-0.12</td>
</tr>
<tr>
<td>Investment</td>
<td>3.67</td>
<td>0.97</td>
<td>0.78</td>
<td>0.18</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>0.87</td>
<td>0.36</td>
<td>-0.01</td>
<td>1.00</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.30</td>
<td>0.60</td>
<td>0.45</td>
<td>0.16</td>
</tr>
<tr>
<td>Private Sector Loans</td>
<td>0.30</td>
<td>0.60</td>
<td>0.45</td>
<td>0.16</td>
</tr>
<tr>
<td>Domestic Bank Deposits</td>
<td>0.55</td>
<td>0.09</td>
<td>0.41</td>
<td>-0.35</td>
</tr>
<tr>
<td>Government Debt</td>
<td>1.04</td>
<td>-0.07</td>
<td>0.30</td>
<td>-0.41</td>
</tr>
<tr>
<td>Default Rate</td>
<td>7.15</td>
<td>0.07</td>
<td>0.10</td>
<td>0.22</td>
</tr>
<tr>
<td>Bank Equity Tax Rate</td>
<td>0.93</td>
<td>-0.46</td>
<td>-0.08</td>
<td>-0.50</td>
</tr>
<tr>
<td>Total Bank Tax Rate</td>
<td>5.77</td>
<td>0.03</td>
<td>0.09</td>
<td>0.16</td>
</tr>
<tr>
<td>Domestic Deposit Tax Rate</td>
<td>0.53</td>
<td>-0.14</td>
<td>-0.50</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Note: All standard deviations are in percentage form except for the tax-rate standard deviations. All data is HP filtered with a penalty parameter of 6.25. These statistics are based on a 400 period simulation as in Chari, Christiano, and Kehoe (1995).
References


69