Optimal Fiscal Policy and the Banking Sector

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Abstract

What should the government’s fiscal policy be when banks hold significant amounts of public debt and the government can default on its debt obligations? This question is addressed using a dynamic general equilibrium model where banks face constraints on their leverage ratios and adjust lending to satisfy regulatory requirements. In response to adverse real shocks, the government subsidizes banks and accelerates bond repayments to sustain private sector lending. When government consumption exogenously increases, however, the government optimally taxes banks and partially defaults on its debt. Debt issuance is procyclical to ensure equilibrium in the deposit market. With an opening of the economy, the government uses less aggressive tax and default policies.

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1 Introduction

Recent historical episodes suggest that governments are willing to expend large amounts of resources in order to support their banking sectors during recessions. According to the *Economist* even as bond spreads between Germany and Spain were reaching euro era records in 2012, Spain’s Prime Minister pledged to not allow any Spanish bank fail and spent $19 billion dollars (close to two percent of Spain’s GDP) bailing out Bankia, a major Spanish bank. Also, IMF (2002) discusses how Ecuador ended up using all of the revenue from its 1999-2000 sovereign default to recapitalize its banking sector (the fiscal benefit of defaulting was $2.3 billion but the fiscal cost bailing out the banking sector was $2.7 billion).

In addition, there have been many examples of governments imposing capital controls during crises periods in an attempt to protect their banking sectors. According to IMF (2002) Ecuador, Russia, Ukraine, and Pakistan all imposed either capital controls or deposit freezes during their sovereign debt crises. In this paper, I present a model in which subsidies to the banking sector and taxes on foreign bank accounts are an optimal response to negative productivity shocks.

This paper examines optimal government tax, bond repayment, and debt issuance policies in a dynamic stochastic general equilibrium model with a banking sector. In the model the banking sector is subject to a constraint on its leverage ratio and so in response to a shock that reduces banks’ equity, banks optimally cut back on lending. Also, banks hold large amounts of government debt and so when the government partially defaults on its bonds, bank equity and private-sector lending both decline. In Section 6.1 of the paper I demonstrate that, because of this, government defaults and bank taxes have
harmful effects on the real economy.\footnote{Kumhoff and Tanner (2005) discuss how banks hold large amounts of domestic government debt, especially in developing countries. The authors argue that the primary cost associated with government default is a deterioration of the domestic banking sector. According to IMF (2002), when the Russian government defaulted on its bonds in 1998, over 30 percent of domestic bank assets were restructured and most of the top 50 Russian banks became insolvent. This lead the cost of capital to increase dramatically and real credit to decline by 12 percent.}

In Section 6.2 of the paper I show that in this environment, the government optimally subsidizes the banking sector in response to negative productivity shocks. However, the government partially defaults on bonds held by banks and taxes the banking sector when revenue needs exogenously increase (perhaps due to war). I also find that the government optimally imposes large taxes on workers’ savings in response to both negative productivity shocks and government consumption increases. These taxes have a negative income effect on workers which leads workers to reduce their consumption and increase their labor supply. Finally, government debt issuance is procyclical in the model. Given the large countercyclical taxes on workers, procyclical debt creation helps the government to balance its budget. Also, procyclical government debt issuance helps to ensure that there is an optimal amount of investment in the private economy.

In Section 7 of the paper I modify the model so that domestic banks must compete with foreign banks for deposits. In this environment the government optimally taxes foreign bank accounts in response to negative productivity shocks and the volatility of government defaults and domestic bank subsidies/taxes declines. However, when government consumption exogenously increases, foreign-deposit tax rates decline. This reduces the incentive for workers to increase their foreign borrowing in response to the government shock.

The Ramsey problem presented in this paper is solved using the primal approach. I assume that the government has access to a commitment technology which allows it to
commit to a set of policies in the initial period. I solve the model both in Dynare and via the discrete state-space method.

1.1 Related Literature

Similarly to this paper, Sosa (2012) and Gennaioli et al. (2010) build models where sovereign default is economically costly because it negatively impacts the banking system and this in turn leads to a decline in private sector credit. These papers also analyze the circumstances under which a government would optimally decide to default on its debt. While both papers provide effective models for explaining how sovereign default can lead to a reduction in private sector credit and both papers are able to match certain aspects of the data, my model offers an improvement in a number of areas. Specifically, unlike Sosa and Gennaioli et al., my model includes capital and an endogenous level of bank equity.\(^2\) Also, I focus on the optimal size of government default over the business cycle and in response to both productivity shocks and government consumption shocks (Sosa and Gennaioli et al. only consider productivity shocks, Gennaioli et al. does not analyze business cycle dynamics, and Sosa does not allow for partial defaults). In addition, unlike Sosa and Gennaioli et al. I allow for the possibility that the government can subsidize, as well as default on, the banking sector and I allow the government to have access to many different tax instruments (all of which are distortionary). Finally, unlike Sosa and Gennaioli et al. I examine optimal government behavior when the economy is open and depositors can move their deposits abroad if the domestic banking sector is providing poor returns.\(^3\)

\(^2\)Both Sosa and Gennaioli et al. do include an endogenous decision to hold government bonds which in turn impacts bank equity. However both models rely on exogenous endowments of banker wealth.

\(^3\)However, it should be noted that I assume the government has access to a commitment technology whereas Sosa and Gennaioli et al. do not.
My results differ sharply from the results of Sosa and Gennaioli et al. Both find that the government optimally defaults when productivity is low whereas I find that the government optimally subsidizes the banking sector and accelerates bond repayments in response to negative productivity shocks. In my model, accelerating bond repayments improves banks’ equity position and this prevents banks from becoming severely leverage-ratio constrained for an extended period of time. This in turn prevents banks from significantly reducing their loans to the private sector and it mitigates the effects of the negative shock. So, of the three papers only this paper can adequately explain why governments often expend vast resources bailing out banks even when their fiscal position is weak.

The model structure is similar to Meh and Moran (2010) who build a model where, due to financial frictions, the quantity of bank capital determines how much banks are able to borrow from depositors and lend. This amount of lending in turn determines how much capital is purchased and how much output is produced. Meh and Moran use their model to analyze how banking sectors can amplify productivity and monetary policy shocks and explain how exogenous shocks to bank equity have negative economic consequences. In contrast, I focus exclusively on optimal government tax, debt, and bond repayment policies in a model where government debt is held by the banking sector. Also, Meh and Moran derive an endogenous capital constraint whereas I rely on an exogenous one. However, Meh and Moran assume that banks are risk neutral and an exogenous percentage of banks exit the economy each period. I assume that banks are risk averse and infinitely lived. This allows for a complex decision by bankers in terms of whether to distribute dividends or retain earnings and build up equity.

Following Stiglitz (1980), Chari and Kehoe (1999) and many others, I solve the model using the primal approach. Lockwood (2010) and others also employ the primal
approach to discuss optimal taxation of the financial sector. Specifically, Lockwood (2010) analyzes optimal taxation of banks in the case where banks supply payment services to agents and in the case where banks act as financial intermediaries between households and firms (i.e. banks monitor firms on behalf of households).\(^4\) However, to the best of my knowledge this is the first paper to analyze optimal taxation of banks in a stochastic general equilibrium setting where banks are required to maintain adequate leverage ratios.

In this paper I present a real-goods economy. This enables me to focus exclusively on the optimal fiscal policy response to shocks. Because of this, the model is well suited for studying countries that are in currency unions like the euro. Dellas, Diba and Loisel (2010) study optimal fiscal and monetary policy in an economy with a banking sector subject to financial frictions. Similarly to this paper, the authors find that when the banking sector experiences an exogenous increase in loan defaults, the government should optimally provide fiscal transfers. However, Dellas, Diba and Loisel’s paper differs from this paper in a number of important respects. First, it presents a monetary economy and assume there are price rigidities in place. Also, unlike this paper, it does not include a capital stock or government debt in their model and all taxes are lump sum rather than distortionary.\(^5\) Finally, financial frictions arise because there is a cost associated with altering bank dividends from their steady-state level (banks prefer to provide a smooth stream of dividends). I assume that financial frictions arise because banks are subject to a constraint on their equity-to-asset ratio.

The rest of the paper proceeds as follows. In Section 2 I describe the basic model. In Section 3 I present two possible scenarios for government policy. In the first, the gov-

\(^4\)Lockwood presents nonstochastic models.
\(^5\)Government bonds are implicitly included in their model in the sense that the central bank buys and sells these bonds in order to conduct monetary policy.
ernment’s bond repayment decision is completely exogenous and stochastic. Sometimes
the government pays creditors slightly more than what they are owed and sometimes it
pays them slightly less. The purpose of this section is to show that government default
is economically costly because it leads to a decline in private sector lending and output.

I then present a scenario where the government faces a standard Ramsey problem: gov-
ernment consumption is exogenous and stochastic, but tax and bond repayment policies
are set optimally. I use this scenario to derive the key model results. Section 4 describes
the solution procedure and Section 5 describes the calibration procedure. Section 6 ex-
plains the results. Section 7 presents the slightly modified small open economy version
of the model. The small open economy model enables me to describe how the results
change when workers have the option of depositing their savings abroad. Section 8 pro-
vides a robustness check for the results and Section 9 concludes. Appendices A and
B provide proofs to propositions stated in the paper. Finally, in Appendix C I describe
how to solve the model using the discrete state-space method and present the results
from using this solution procedure.

2 Model

Following Chari, Christiano, and Kehoe (1995) and others I adopt the following no-
tation. \( s_t \) represents the realization of an exogenous event in period \( t \) and \( s^t \) represents
the history of events leading up to and including \( s_t \) (i.e. \( s^t = (s_0, ..., s_t) \)). For a given
variable, \( x \), \( x(s^t) \) represents the value of \( x \) as a function of history \( s^t \). In the base-
line (non-discrete state-space) model I assume there are an infinite number of possible
realizations for \( s_t \) in each period.
2.1 Workers

Workers in the model supply labor, consume, and save. Workers maximize the following objective function:

\[ \sum_{t,s} \beta^t \pi(s^t) U^W(c(s^t), l(s^t)), \]

(1)

where \( c \) is consumption of private goods, \( l \) is the quantity of labor supplied, \( \beta \) is the discount rate, \( \pi(s^t) \) is the probability of history \( s^t \) occurring and

\[ U^W(c(s^t), l(s^t)) = \frac{c(s^t)^{1-\sigma}}{1-\sigma} + \xi [1 - l(s^t)]. \]

(2)

Workers earn income by supplying labor and earning wage rate \( w \) and by earning interest on their savings \( a \). They have one unit of time to divide between leisure and work. Each period labor earnings are taxed at rate \( \tau^l \). Also, all money that workers save is deposited into a savings account where it earns a gross interest rate \( R \). This interest income is taxed at rate \( \tau^a \). Workers’ budget constraint is the following:

\[ [1 - \tau^l(s^t)] w(s^t) l(s^t) + [1 - \tau^a(s^t)] R(s^{t-1}) a(s^{t-1}) = c(s^t) + a(s^t). \]

(3)

The intertemporal and intratemporal conditions are the following:

\[ c(s^t)^{-\sigma} = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) [1 - \tau^a(s^{t+1})] R(s^{t+1}) c(s^{t+1})^{-\sigma}, \]

(4)

\[ [1 - \tau^l(s^t)] w(s^t) c(s^t)^{-\sigma} = \xi. \]

(5)
2.2 Banks

Banks have liabilities, assets, and equity. Bank liabilities are the deposits that they accept from workers. Bank assets are equal to the value of loans to producers in the private sector and loans to the government. Each period banks lend firms capital \( k \) which firms then use for production. Loans to the government occur through the purchase of government bonds \( b \). Bank equity, \( n \), is equal to \( k + b - a \) and will be described further below. The objective of banks is to maximize the following:

\[
\sum_{t=0}^{\infty} \beta^t \pi(s^t) U^B(m(s^t)),
\]

(6)

where \( m \) is banker consumption and

\[
U^B(m(s^t)) = \frac{m(s^t)^{1-\sigma}}{1 - \sigma}.
\]

(7)

The representative bankers’ budget constraint is the following:

\[
m(s^t) + [1 + \gamma + \tau^e(s^t)]n(s^t) = R^k(s^t)k(s^t-1)[1 - \tau^k(s^t)] + R^b(s^t-1)[1 - d(s^t)]b(s^t-1) - R(s^t-1)a(s^t-1).
\]

(8)

In Equation (8) the right-hand side is the gross return on bank equity in period \( t \). \( R^k \) is the gross return on loans to producers, \( \tau^k \) is a tax on banks’ loan revenue, and \( \tau^e \) is a tax on banks’ equity. \( d \) is the default rate on government bonds and \( R^b \) is the gross return on government bonds that investors would receive if there was no default (to be explained in more detail later).
In Equation (8), $\gamma$, represents a cost that banks face for having large amounts of equity. This cost gives banks an added incentive to pay dividends instead of reinvesting their return on equity (i.e. allocating resources to $m$ instead of $n$). This cost is motivated by the idea that as bank equity increases, so do principle-agent problems between bank owners and managers. For example, as equity increases managers may have more incentive and ability to devote resources to pet projects and personal perks instead of to profit-maximizing investments. Many economists believe that principle-agent costs such as these can explain why firms (not just banks) pay dividends even though the effective capital-gains tax rate is lower than the effective dividend tax rate.\(^6\) $\gamma$ is very important for solving the model. In the Ramsey problem described below, if $\gamma$ did not exist then the government would set bank equity very high and impose a complex set of subsidies, taxes, and defaults on banks. This would prevent banks from ever becoming equity-ratio constrained (described next) without allowing bankers’ consumption to become large.

I assume that banks are subject to an equity-ratio constraint that looks as follows:

$$\mu n(s^t) \geq k(s^t),$$  \hspace{1cm} (9)

where $\mu$ is a parameter that determines how much equity banks must hold to satisfy regulators. Government debt, $b$, is not included as an asset in this equity-ratio constraint since regulators assign a zero risk weighting to government debt. Also, setting up the equity-ratio constraint like this allows me to use the primal approach when solving the Ramsey problem described below. Bankers maximize lifetime utility by lending capital to the point where:

\(^6\)See Gruber (2010) for a discussion of this issue.
\[ \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t)U^B_m(s^{t+1})R^k(s^{t+1})(1 - \tau^k(s^{t+1})) = \]
\[ \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t)U^B_m(s^{t+1})R(s^t) + \chi(s^t), \]  
where \( \chi \) is the Lagrange multiplier on the equity-ratio constraint. In Equation (10) the left-hand side is the expected marginal benefit of lending and the right-hand side is the expected marginal cost. Bankers maximize lifetime utility by purchasing government bonds to the point where:

\[ \sum_{s^{t+1}} \pi(s^{t+1} | s^t)U^B_m(s^{t+1})(1 - d(s^{t+1}))R^b(s^t) = \]
\[ \sum_{s^{t+1}} \pi(s^{t+1} | s^t)U^B_m(s^{t+1})R(s^t). \]  

Finally, bankers maximize lifetime utility by purchasing \( n \), i.e. reinvesting their return on equity, to the point where:

\[ U^B_m(s^t)(1 + \gamma + \tau^e(s^t)) = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t)U^B_m(s^{t+1})R(s^t) + \mu \chi(s^t). \]  

Equation 12 is the Euler equation for bankers.

### 2.3 Firms

Firms produce output, \( Y_t \), using labor and capital according to the following production function:
\[ Y(s^t) = z(s^t)k(s^{t-1})^\alpha l(s^t)^{1-\alpha}, \]  

(13)

where \( z(s^t) \) is total factor productivity (TFP) and it has a mean of one. TFP evolves according to the following equation:

\[ \log(z(s^t)) = \rho^z \log(z(s^{t-1})) + e^z(s_t), \]  

(14)

where \( e^z(s_t) \) is an exogenous and i.i.d. random shock with a mean equal to zero. Firms’ profit is equal to \( Y(s^t) + (1 - \delta)k(s^{t-1}) - w(s^t)l(s^t) - R^k(s^t) \) where \( \delta \) is the depreciation rate of capital. The markets for labor and capital are competitive and so input prices are equal to their marginal products. Specifically,

\[ w(s^t) = (1 - \alpha)z(s^t)k(s^{t-1})^\alpha l(s^t)^{-\alpha} \]  

(15)

and

\[ R^k(s^t) = \alpha z(s^t)k(s^{t-1})^{\alpha - 1}l(s^t)^{1-\alpha} + (1 - \delta). \]  

(16)

### 2.4 Government

The government earns income from taxing workers’ labor earnings and deposit-interest income and from taxing bankers’ equity and private loan returns. The government also raises money by issuing bonds. All revenue is used for government consumption and for paying interest and principal on previously issued bonds. Finally, the government can reduce its need to raise revenue by defaulting on previously issued bonds. The government’s budget constraint is the following:
\[
\begin{align*}
\tau^l(s^t)w(s^t)l(s^t) + \tau^a(s^t)R(s^{t-1})a(s^{t-1}) + \tau^k(s^t)R^k(s^t)k(s^{t-1}) \\
\tau^e(s^t)n(s^t) + b(s^t) &= g(s^t) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}),
\end{align*}
\] (17)

where \(g(s^t)\) is government consumption. In Section 3, I present two scenarios. In the first scenario, the government’s tax and default policies are exogenous. The purpose of this scenario is simply to demonstrate that in the model an unexpected government default leads to a ‘financial crisis’ and is associated with a contraction in bank lending, and output. In the second scenario, tax and default policies are set optimally. Given that default is costly, this scenario demonstrates how an optimizing government would alter debt, bond repayment, and tax policies in response to economic shocks.

### 2.5 Equilibrium

**Proposition 1:** Given the budget constraints of the representative worker, representative banker, and the government, the aggregate resource constraint is the following:

\[
\begin{align*}
Y(s^t) + (1 - \delta)k(s^{t-1}) &= c(s^t) + k(s^t) + m(s^t) + g(s^t) \\
&+ \gamma n(s^t).
\end{align*}
\] (18)

Proof: See Appendix 1.

A government policy is a sequence of \(\tau^l(s^t), \tau^a(s^t), \tau^k(s^t), d(s^t), \tau^e(s^t),\) and \(R^b(s^t)\) for every \(s^t\). An allocation is a sequence of \(l(s^t), a(s^t), k(s^t), c(s^t), m(s^t), n(s^t), b(s^t)\) and \(g(s^t)\) for every \(s^t\). And a price system is a sequence of \(w(s^t), R(s^t),\) and \(R^k(s^t)\) for every \(s^t\).
Similarly to Chari, Christiano, and Kehoe (1995), a competitive equilibrium is defined as an allocation, government policy, price system, and initial values $k_{-1}, a_{-1}, n_{-1}, R_{-1}, R^b_{-1}$, and $b_{-1}$ that meet the following criteria: the allocation maximizes workers’ lifetime utility, (1), subject to (3), and the allocation maximizes bankers’ lifetime utility, (6), subject to (8) and (9); wages are given by (15) and the return on capital is given by (16); and the government satisfies its budget constraint, (17), in every period.

3 Government Policy

3.1 Exogenous Government Policy

In this section I assume that government policy is exogenous and history independent. This allows me to clearly demonstrate that the model matches a key stylized fact: government default leads to a significant decline in bank lending and output. This stylized fact is supported by empirical research from Borensztein and Panizza (2009). The authors find that sovereign default is associated with a decline in output growth and an increase in the probability of a banking crisis. In the next section I derive optimal government policies given the costs of default.

My primary interest is the impact of government default on the model economy. So, for simplicity I assume that $\tau^a, \tau^e, \text{ and } \tau^k$ are zero in all periods. $\tau^l$ is constant and set so that in the steady state, when $d$ equals zero and $z$ equals one, the government’s budget is balanced. Also I assume that the government adopts a simple decision rule for how to respond to a deficit (surplus):

$$\bar{g} - g(s^t) = X[\bar{g} + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - \bar{\tau}^l w(s^t)l(s^t) - \bar{b}]$$ (19)
\[ b(s^t) - \bar{b} = (1 - X)[\bar{g} + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - \bar{\tau}^l w(s^t)l(s^t) - \bar{b}] \quad (20) \]

These equations show that in response to an increase in interest expenses or a decline in tax revenue, a share \( X \) of the adjustment needed to balance the budget occurs through a decrease in spending and a share \((1 - X)\) of the adjustment occurs through an increase in debt. For simplicity, I make \( d \), the variable of interest, an i.i.d. random variable with a mean of zero. This implies that sometimes the government partially defaults and sometimes the government pays banks more than expected. The latter case can be thought of as a subsidy to the banking sector.

I set \( X \) to 0.1. When \( X \) is large, shocks to \( d \) are associated with large shocks to government consumption. Because of this, the negative impact that default has on the banking sector is offset to some extent by the expansionary impact that default has on government consumption. The purpose of this section is to demonstrate how default can lead to a reduction in economic activity through its impact on the financial sector and so, for ease of exposition, it makes sense to set \( X \) to a low number. However, when \( X \) is too low \( b \) does not converge to its steady-state value in response to a \( d \) shock. Results for this model setup are described in Section 6.1.

### 3.2 Optimal Government Policy

In this section I assume that the government’s goal is to maximize the following social welfare function:
\[ \sum_{t,s_t} \beta^t \pi_t(s_t)[U^W(c(s_t), l(s_t)) + \theta U^B(m(s_t))], \tag{21} \]

where \( U^W(c(s_t), l(s_t)) \) is given by Equation (2) and \( U^B(m(s_t)) \) is given by Equation (7). \( \theta \) is an exogenous parameter and it describes the relative weight of bankers in the government’s social welfare function. One would expect \( \theta \) to be large if, for instance, there are many citizens who work as bankers or if banks are major political contributors to the government.

In the last section, all government-related variables were exogenous. Now however, only government consumption is exogenous and all other government-related variables are set optimally. When the model is solved in Dynare, I assume government consumption evolves according to:

\[ \log(g(s_t)) = (1 - \rho^g) \log(\bar{g}) + \rho^g \log(g(s_{t-1}^s)) + e^g(s_t). \tag{22} \]

When the model is solved using the discrete-state method, I assume that government consumption follows a two-state Markov process.

Following Chari, Christiano, and Kehoe (1995) I solve this Ramsey optimal taxation problem using the primal approach.\(^7\) I assume the government has access to a commitment technology where at time zero it can commit to a policy for the rest of time. Once the government announces its policy, workers and bankers adopt allocation rules. These allocation rules determine allocations based on the announced government policy (see Chari and Kehoe (1999)).

Following Chari and Kehoe (1999), a Ramsey equilibrium is defined as a policy,
allocation, price system, and initial values \(k_{-1}, a_{-1}, n_{-1}, R_{-1}, R^b_{-1}\), and \(b_{-1}\), such that the government’s policy maximizes the social welfare function, Equation (21), subject to the aggregate resource constraint, Equation (18); and the requirements of a competitive equilibrium are satisfied.

**Proposition 2:** (i) The allocation implied by the Ramsey equilibrium maximizes (21), subject to the aggregate resource constraint in each period, Equation (18), the equity-ratio constraint in each period, Equation (9), and the following two implementability constraints:

\[
\sum_{t,s} \beta^t \pi^t(s^t)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)] = U^W_c(s_0)R^{-1}_{-1}[1 - \tau^a(s_0)]a_{-1},
\]

and

\[
\sum_{t,s} \beta^t \pi^t(s^t)U^B_m(s^t)m(s^t) = U^B_m(s_0)[[1 - \tau^k(s_0)] R^k(s_0)k_{-1} + [1 - d(s_0)] R^{-1}_{-1}b_{-1} - R_{-1}a_{-1}].
\]

(ii) Given the fact that an allocation satisfies the above-mentioned constraints, it is possible to construct a government policy and a price system such that the allocation, government policy, and price system satisfy the definition of a competitive equilibrium.\(^8\)

**Proof:** See Appendix 2.

Let \(\Lambda(s^t)\) be the Lagrange multiplier on the resource constraint for history \(s^t\), \(\chi^G(s^t)\) be the Lagrange multiplier on the equity-ratio constraint, \(\Phi\) be the the Lagrange multiplier on Equation (23), and \(\Gamma\) be the the Lagrange multiplier on Equation (24). Similarly

\(^8\)This proposition is similar to the ones presented in Chari, Christiano, and Kehoe (1995) and Chari and Kehoe (1999).
to Chari, Christiano, and Kehoe (1994) the government’s maximization problem can be written as:

$$\max \sum_{t,s} \beta^t \pi(s^t) W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma)$$

$$+ \Lambda(s^t) [Y(s^t) + (1 - \delta) k(s^{t-1}) - c(s^t) - k(s^t) - m(s^t) - g(s^t) - \gamma n(s^t)]$$

$$+ \chi^G(s^t) [\mu n(s^t) - k(s^t)]$$

$$- \Phi U_c^W(s_0)[1 - \tau^c(s_0)] R_{-1} a_{-1}$$

$$- \Gamma U_m^B(s_0)[1 - \tau^m(s_0)] R_{-1}^b + R_{-1}^b [1 - d(s_0)] b_{-1} - R_{-1} a_{-1}$$

(25)

where

$$W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) = [U_c^W(c(s^t), l(s^t)) + \theta U_m^B(m(s^t))]$$

$$+ \Phi U_c^W(s^t)c(s^t) + U_l^W(s^t)l(s^t)] + \Gamma U_m^B(s^t)m(s^t)$$

(26)

Results for this scenario are described in Section 6.2

4 Solution Procedure

4.1 Solution Procedure for the Exogenous Policy Model

To solve this model I use a second-order Taylor approximation around the steady state and solve using the standard perturbation method (in Dynare). However, this method is unable to solve models with occasionally binding constraints and so I replace the inequality constraint, Equation (9), with a penalty function similarly to Kim, Koll-
man, and Kim (2009). Specifically I change the representative banker’s utility function to look as follows:

\[ U^B(m(s^t), n(s^t), k(s^t)) = \frac{m(s^t)^{1-\sigma}}{1-\sigma} + \phi \log(\mu n(s^t) - k(s^t)) \]  

(27)

### 4.2 Solution Procedure for the Optimal Policy Model

Given the nature of the problem, the decision rules in period \( t = 0 \) are different than in period \( t > 0 \). Specifically, there is an incentive for the government to heavily tax deposits and impose a large default on its bonds in period zero. The reason for this is because in the initial period deposits and government debt are inelastically supplied, so taxation/default on these assets acts as a non-distortionary tax (see Chari and Kehoe (1999)). Due to this, economists usually place some sort of restriction on time-zero policies. Following Benigno and Woodford (2006), I put restrictions on the right-hand sides of (23) and (24) rather than on initial tax/default rates per se. The restrictions are as follows:

\[ H^W = U^W_c(s^0)R_{-1}[1 - \tau^a(s_0)]a_{-1}, \]  

(28)

and

\[ H^B = U^B_m(s^0)[\tau^k(s_0)R^k(s_0)k_{-1} + R^b_{-1}[1 - d(s_0)]b_{-1} - R_{-1}a_{-1}], \]  

(29)

where \( H^W \) and \( H^B \) are parameters. I set \( H^W \) and \( H^B \) to the level that would prevail when all variables are at their steady-state values in a non-stochastic economy with \( d = \tau^a = \tau^c = \tau^k = 0, \bar{b} \) and \( \bar{g} \) equal to their calibrated steady-state values as described
below, and \( z \) set to one.\(^9\) I solve this model using the standard perturbation method as well as the discrete state-space method. More details about how the model can be solved using the latter method are discussed in Appendix C.

## 5 Calibration

In this section I focus on the calibration procedure that is employed when the model is solved in Dynare. I save details specific to the calibration of the discrete-state space version of the model for Appendix C. The only parameters that differ across solution methods are those relating to the stochastic processes of \( g \) and \( z \).

Table 1 lists the values for the calibrated parameters. For \( \alpha \) and \( \xi \), I use the values given in Hansen (1985). I set \( \sigma \) to 1.5 which is within the standard range of estimated values as discussed by Mehra and Prescott (1985). Values for \( \beta \) and \( \delta \) come from Chari, Christiano, and Kehoe (1995). These values are calibrated to match the moments of annual data.

To calibrate \( \gamma \), I use results from a study of agency costs in small businesses by Ang, Cole, and Lin (2000). The authors find that in firms where the primary owner owns 100 percent of the firm and also serves as the manager, the operating expense to sales ratio is 0.464. However in firms where no owner or family owns more than 50 percent of the firm and the firm is managed by an outsider, the operating expense to sales ratio is 0.553. This suggests that agency costs make up 8.9 percent of sales for firms in the latter scenario. The authors’ data also indicates that the average sales to assets ratio is 0.0465 which, assuming for simplicity that the sales to assets ratio is constant across firms with different management/ownership structures, suggests that agency costs make

\(^9\)Calculating the steady-state involves using the worker and banker budget constraints to set \( \bar{m} \) and \( \bar{c} \) once \( k/l \) and \( n/l \) have been solved for.

20
<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.97</td>
</tr>
<tr>
<td>Capital Exponent</td>
<td>$\alpha$</td>
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</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta$</td>
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<tr>
<td>Utility from Leisure Parameter</td>
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<tr>
<td>Relative Importance of Bankers</td>
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<tr>
<td>Banker Risk Aversion</td>
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</tr>
<tr>
<td>Capital Requirement Parameter</td>
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</tr>
<tr>
<td>Technology Autocorrelation Parameter</td>
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</tr>
<tr>
<td>Government Autocorrelation Parameter</td>
<td>$\rho_g^g$</td>
<td>0.80</td>
</tr>
<tr>
<td>Standard Deviation of Productivity Shocks</td>
<td>$\sigma_z^z$</td>
<td>0.018</td>
</tr>
<tr>
<td>Standard Deviation of Government Shocks</td>
<td>$\sigma_g^g$</td>
<td>0.043</td>
</tr>
<tr>
<td>Principle-Agent Cost Parameter</td>
<td>$\gamma$</td>
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</tr>
<tr>
<td>Government Debt* (% of annual steady-state GDP)</td>
<td>$\bar{b}/\bar{Y}$</td>
<td>47%</td>
</tr>
<tr>
<td>Government Consumption* (% of quarterly steady-state GDP)</td>
<td>$\bar{g}/\bar{Y}$</td>
<td>25%</td>
</tr>
<tr>
<td>Labor-Tax Rate*</td>
<td>$\bar{\tau}^l$</td>
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</tr>
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<td>Deposit-Tax Rate*</td>
<td>$\bar{\tau}^a$</td>
<td>0</td>
</tr>
<tr>
<td>Bank-Equity-Tax Rate*</td>
<td>$\bar{\tau}^e$</td>
<td>0</td>
</tr>
<tr>
<td>Foreign Deposit Adjustment Parameter (Optimal-Policy Model)</td>
<td>$\psi$</td>
<td>0.08</td>
</tr>
</tbody>
</table>

* Refers to steady-state values.

All data is hp-filtered with a penalty parameter of 6.25.
up 0.004138 percent of firms’ assets.\textsuperscript{10} Since 1988 the average assets-to-equity ratio at commercial banks has been 11.49. This suggests that agency costs make up roughly 0.048 percent of banks’ equity. In the model agency costs ($\gamma n$) divided by equity ($n$) equals $\gamma$ and so I set $\gamma$ to 0.048 in the model.

I use annual U.S. data over the period 1947 to 2012 from the Bureau of Economic Analysis to calibrate the stochastic process for $g$ and $z$ and to calibrate the steady-state value of $g$. Specifically, I use data on “government consumption expenditures and gross investment,” to calibrate government consumption in the model. This includes expenditure and investment at the federal and state and local levels. The steady-state value of $g$ is calibrated so that, in the steady-state, government consumption divided by output equals the average value of this ratio in the data over the sample period.

To calibrate $\rho^z$ and $\sigma^z$, I used employment data from the Bureau of Labor Statistics and capital stock and output data from the Bureau of Economic Analysis. Specifically, I assumed that output evolves according to:

$$Y_t = \exp(g t) z_t k_t^{\alpha} l_t^{1-\alpha},$$

where $\alpha$ is equal to 0.36. This is the value used in Hansen (1985) based on estimates of the share of total income that accrues to labor. I then estimated the following equation using least squares:

$$\Delta \log(Y_t) - \alpha \Delta \log(k_t) - (1-\alpha) \Delta \log(l_t) = \hat{\rho} + \epsilon_t$$

\textsuperscript{10}The authors actually find that firms that are managed by their owners and where the primary owner owns 100 percent of the firm have higher sales to assets ratios. However, to keep things simple in the model I assume that agency costs only impact the expense side of firms’ net income and so I ignore this result.
and set $\Delta \log(z_t)$ equal to $\epsilon_t$. I then used this to form a series for $z_t$ and ran the following regression:

$$\log(z_t) = \hat{\phi}_0 + \hat{\phi}_1 \log(z_{t-1}) + \epsilon_t^z. \quad (32)$$

Finally I used the standard deviation of $\epsilon^z$ to get an estimate for the standard deviation of the productivity shock in the model, $\sigma^z$, and the coefficient on the lag term in Equation (32) to get an estimate for $\rho^z$ in the model. I calibrated $\rho^g$ and $\sigma^g$ by estimating the equation,

$$\log(g_t) = \hat{\vartheta}_0 + \hat{\vartheta}_1 t + \hat{\vartheta}_2 \log g_{t-1} + \epsilon_t^g. \quad (33)$$

Then I set $\rho^g$ equal to $\hat{\vartheta}_2$ and set $\sigma^g$ equal to the standard deviation of $\epsilon^g$. To calibrate the steady-state level of government debt I use data from the IMF World Economic Outlook Database on U.S. general government net debt over the period 1980 to 2012.\footnote{This is as far back as the data goes.}

Recall that $\mu$ is a parameter that dictates how much equity banks must hold relative to private-sector loans and $\theta$ is a parameter that dictates how much weight the government puts on bankers’ utility. In the U.S. banks must hold eight cents in Tier 2 capital for every dollar they hold in risk-weighted assets and so $\mu$ is set to 12.5. The steady-state ratio of capital to banker equity is determined entirely by $\mu$ and because of this, and the aforementioned restrictions on time-zero policies, $\theta$ does not have any impact on steady-state allocations. However, as discussed later, $\theta$ does influence the standard deviation of policy variables, prices, and allocations. In the baseline model, $\theta$ is set to equal the steady-state ratio of worker consumption plus worker deposits divided by banker consumption plus bank equity.
In the model with exogenous government default policies, $\phi$, the penalty-function parameter used to approximate the occasionally binding equity-ratio constraint is set so that in the steady-state bankers choose to hold 50 percent more equity than the minimum required amount (i.e. the equity to risk-weighted assets ratio is 0.12 instead of the legally required 0.08).\textsuperscript{12} Finally, in the small open economy model presented below, the adjustment cost parameter, $\psi$, is set to match the standard deviation of net exports divided by output in annual US data from 1947 to 2011.\textsuperscript{13}

6 Results

6.1 Results for Exogenous Policy

Figure 1 shows the impulse responses associated with a government default shock. In the figure, the government has paid creditors (banks) only 99 percent of what it owes them. Default has an immediate and severe impact on investment. In response to the default, banks reduce lending in order to rehabilitate their equity-to-asset ratios. This in turn leads to a decline in the capital stock and output. However, the decline in the capital stock leads to an increase in the marginal product of capital which leads to large returns for bankers on their remaining loans. These positive returns gradually improve bankers’ net worth which allows for a gradual increase in lending and output.

I solve the model using a second-order Taylor approximation, and so amplifying the

\textsuperscript{12} The penalty function, model structure and solution method are such that the range of possible values for the steady-state equity to risk-weighted assets ratio is restricted. That being said, Sonali and Sy (2012) study a sample of over 700 banks from over 30 countries and find that the average total capital to risk-weighted assets ratio was 0.1447 in 2006 and 0.1496 in 2010. This is relatively close to the steady-state value of 0.12.

\textsuperscript{13} $\psi$ is set to different values in the exogenous default model and the Ramsey model. In both models $\psi$ is set to match the standard deviation of net exports divided by GDP (when neither variable is hp-filtered). In the data this standard deviation is 1.6 percent.
size of the default has nonlinear impacts on the results. For example, in an economy with no productivity shocks increasing the standard deviation of \( d \), the default rate, from one to two percent causes the standard deviation of output to increase by 0.037 percentage points but increasing the standard deviation of \( d \) from two to three percent causes the standard deviation of output to increase by an additional 0.062 percentage points.\(^{14}\)

Increasing the standard deviation of \( d \) surprisingly leads to higher expected lifetime utility for bankers. The reason is because when the standard deviation of \( d \) increases so does the risk premium that the government must pay bankers in order for bankers to buy government bonds. For example, when the standard deviation of \( d \) increases from zero to two percent the average risk premium increases from zero to five basis points. Some of the money required to pay for these risk premiums is raised by reducing government consumption. In the model government consumption does not yield any utility to bankers or workers and so a reduction in government consumption allows for an increase in banker and worker consumption without having any offsetting negative consequences. When the standard deviation of \( d \) increases, there is also a decrease in both worker and banker utility because the volatility of both worker and banker consumption increase. For example, when the standard deviation of \( d \) increases from zero to two percent the standard deviation of banker consumption increases by 0.02 percentage points and the standard deviation of worker consumption increases by 0.05 percentage points. For bankers, the first effect overwhelms the second and there is a net increase in bankers’ average utility when the standard deviation of \( d \) increases. For workers the effect of a small decline in government consumption is not enough to make up for the increase in consumption volatility and average utility decreases slightly.

\(^{14}\)With the aforementioned penalty function in place, the model is unable to handle calibrations where the standard deviations of \( d \) is equal or larger than five percent.
Figure 1: Exogenous Policy Model Response to a One Percent Government Default
6.2 Results for Optimal Policy

Before discussing the results, it is important to note that with the utility function from Equation (2) in place, the tax rate on labor is completely constant (impulse response function not shown). To see this note that the intratemporal condition for the government is as follows:

\[(1 − \alpha)z(s^t)k(s^{t−1})^\alpha l(s^t)^{−\alpha}c(s^t)^{−\sigma} = \xi(1 + \Phi).\]  \hspace{1cm} (34)

After noting that \(w\) is equal to the left-hand side of (34), (5) can be written as:

\[(1 − \alpha)z(s^t)k(s^{t−1})^\alpha l(s^t)^{−\alpha}c(s^t)^{−\sigma} = \frac{\xi}{[1 − \tau^a(s^t)]}.\] \hspace{1cm} (35)

Comparing (34) and (35) it is clear that the optimal policy is for the government to keep tax labor rates constant and equal to \(\Phi/(1 + \Phi)\). This is very similar to Chari and Kehoe (1995) who find that the optimal standard deviation of labor tax rates is extremely small.\(^{15}\)

As discussed in Appendix B, due to the indeterminacy in the model, there are some variables that the theory does not uniquely pin down. However, the theory does uniquely determine the government’s optimal debt level, and the optimal tax rate on equity. Also the model pins down the after-tax return on deposits in each state and the after-tax net income for bankers in each state.\(^{16}\) In other words,

\[ [1 − \tau^a(s^t)]R(s^{t−1})a(s^{t−1}) \]

\(^{15}\)In simulations where the productivity shock is calibrated to U.S. data, Chari and Kehoe (1995) show that the standard deviation of labor tax rates is only 0.1 percent over the business cycle.

\(^{16}\)In the international model presented below, the optimal tax rate on foreign assets is determinate only because I impose the restriction that foreign-asset tax rates are set one period in advance.

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are pinned down in each state. In order to turn these variables into something that resembles a tax rate, I compare what workers and bankers actually receive to what they would have hypothetically received in an economy that lacks deposit taxation, capital taxation, equity taxation, and defaults. In this economy, according to Equation (4), workers would receive

\[ \tilde{\tau}_{a}(s^t) = \tilde{R}(s^t) - \left[1 - \frac{c(s^t) - \sigma}{\beta \sum_{s'} \pi(s^t+1 | s') c(s^t+1) - \sigma} \right] \tilde{R}(s^t-1) \tag{38} \]

in an economy without deposit taxation. Therefore, I define

\[ \tilde{\tau}_{a}(s^t) = \frac{\tilde{R}(s^t-1) - [1 - \tau_{a}(s^t)]R(s^t-1)}{\tilde{R}(s^t-1)}. \tag{39} \]

In the steady state it must be the case that \( \tilde{\tau}_{a}(s^t) \) equals zero. Similarly, in an economy without deposit taxes, capital taxes, equity taxes or defaults, bankers would earn a gross rate of return of:

\[ \tilde{\rho}(s^t) = \frac{[\tilde{R}(s^t) - \tilde{R}(s^t-1)]k(s^t-1) + \tilde{R}(s^t-1)n(s^t-1)}{[1 + \gamma] n(s^t-1)}. \tag{40} \]

on their equity. However, because these taxes are in fact in place, the return on equity ends up being:
\[ R^e(s^t) = \frac{[R^k(s^t)[1 - \tau^k(s^t)]k(s^{t-1}) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) - R(s^{t-1})a(s^{t-1})]}{[1 + \gamma + \tau^e(s^{t-1})]n(s^{t-1})}. \]  

(41)

Then I define:

\[ \tilde{d}(s^t) = \frac{\hat{R}^e(s^t) - R^e(s^t)}{\hat{R}^e(s^t)}. \]

(42)

As with \( \tilde{\tau}^a(s^t) \), in the steady state \( \tilde{d}(s^t) \) equals zero.

\( \tau^e(s^t) \) and \( \tilde{d}(s^t) \) can be used to define a measurement for the total effective tax burden on bankers’ equity. I accomplish this by dividing the total burden on bankers due to taxation in a given period by the total income bankers would have received if there were no finance-related taxation.\(^{17}\)

\[ \tau^B(s^t) = \frac{\tau^e(s^t)n(s^t) + \tilde{d}(s^t)[R^k(s^t) - \hat{R}(s^{t-1})]k(s^{t-1}) + \hat{R}(s^{t-1})n(s^{t-1})]}{[R^k(s^t) - \hat{R}(s^{t-1})]k(s^{t-1}) + \hat{R}(s^{t-1})n(s^{t-1})}. \]

(43)

Table 2 shows the business-cycle statistics for the model. These statistics indicate that the government actively employs taxes and subsidies on both workers and bankers. Bank tax rates are procyclical but deposit tax rates are modestly countercyclical and the government issues debt procyclically. Perhaps the most surprising result is the large standard deviation of the government’s debt level, at more than five times the percentage standard deviation of output.\(^{17}\)

\(^{17}\)The idea to define taxes in this way was inspired by Chari and Kehoe (1999).
### Table 2: Simulated Business-Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Deviation</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(x)/\sigma(Y)$</td>
<td>$\rho(x,Y)$</td>
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<tr>
<td>Private Consumption</td>
<td>0.59</td>
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<tr>
<td>Investment</td>
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<td>Government Consumption</td>
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<tr>
<td>Bank Equity</td>
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<tr>
<td>Private Sector Lending</td>
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<tr>
<td>Domestic Bank Deposits</td>
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</tr>
<tr>
<td>Government Debt</td>
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<tr>
<td>Default Tax Rate ($\tilde{d}$)</td>
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<tr>
<td>Bank Equity Tax Rate</td>
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<tr>
<td>Total Bank Tax Rate ($\tau^B$)</td>
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<td>0.80</td>
</tr>
<tr>
<td>Domestic Deposit Tax Rate ($\tilde{\tau}^a$)</td>
<td>1.29</td>
<td>-0.37</td>
</tr>
</tbody>
</table>

Note: All standard deviations are in percentage form except for the foreign deposit and tax-rate standard deviations.

Figure 2 shows the impact of a one standard deviation negative productivity shock. Optimally the government taxes deposits most heavily in the period of the shock. Front-loading this tax enables the government to collect revenue while minimizing distortions to the saving/consumption decision. Also, increasing deposit tax rates is optimal because it transmits the shock’s negative wealth effect to workers. This negative wealth effect incentivizes workers to reduce their consumption and increase their labor supply even though, all else constant, the TFP shock reduces wages.\footnote{Dellas et al. (2010) also find that imposing negative wealth effects is the optimal response to certain types of financial shocks.}

Interestingly, the government finds it optimal to subsidize banks in response to a negative shock. The government achieves these subsidies through a decrease in $\tilde{d}(s^t)$ (the default variable) in the period of the shock and through negative equity taxation.
Figure 2: Endogenous Policy Model Response to a One Standard Deviation Negative Productivity Shock

Notes: All tax rates are absolute deviations from their steady-state values and all quantities are percentage deviations from their steady-state values.
after the shock. These subsidies help banks to muddle through the upcoming economic downturn without becoming capital-constrained. Conversely, the government finds it optimal to increase $\tilde{d}(s^t)$ and tax bank equity in response to positive productivity shocks.

Comparing the government and the representative banker’s first-order conditions can give insight into the reasons behind these policy responses. The government’s first-order condition with respect to bank equity is the following:

$$\gamma U^W_c(s^t) = \mu \chi^G(s^t), \quad (44)$$

where $\chi^G$ is the Lagrange multiplier associated with the equity-ratio constraint in the government’s Ramsey problem. The bankers’ first-order condition is the following:

$$U^B_m(s^t)[1 + \gamma + \tau^e(s^t)] = \beta \sum_{s^{t+1}} \pi(s^{t+1} | s^t) U^B_m(s^{t+1})R(s^t) + \mu \chi(s^t). \quad (45)$$

As the government’s first-order condition shows, the government prefers to set $n(s^t)$ exactly at the point where equity-ratio constraint binds at all times. This minimizes principle-agent costs associated with having large amounts of bank equity in the economy and at the same time satisfies the equity-ratio constraint. However, holding tax rates constant, banks may prefer to let bank equity build relative to private sector loans during good times when interest rates are high and the opportunity cost of accepting deposits is large. Alternatively, banks would let their equity fall relative to private-sector loans and allow themselves to become credit constrained during bad times when interest rates are low. So, the government adjusts tax rates and bond repayments to make its interest coincide with bankers’ interest. The government taxes banks during good times in order to discourage what it views as an unnecessary build up of equity. And the government subsidizes bank equity during bad times to ensure that equity-constrained
banks do not curtail private-sector lending too severely.  

Surprisingly, the government reduces its debt load in response to the negative shock. The reason is that during the downturn workers want to reduce their savings (to smooth consumption) at a sharper rate than the government wants to reduce investments in the private economy. To accommodate workers the government optimally reduces government bonds in the economy. In practice, one way the government may introduce and remove bonds from the economy is through central bank ‘quantitative easing’ programs. Also, the reduction in debt reflects the increase in revenue associated with the increase in deposit tax rates. The positive correlation between debt and output contrasts strongly with the seminal work of Barro (1979) who finds that the government should optimally reduce tax revenue during a downturn and it should finance this reduction by issuing debt.

Figure 3 shows the optimal government response to an exogenous one standard deviation increase in government consumption. Interestingly, the government finds it optimal to increase default/tax rates on both bankers and workers in response to the shock. Again, these taxes are front-loaded to minimize the distorting impact on saving and investment. Also, once again the deposit tax has a negative wealth effect on workers and this leads workers to increase their labor supply and reduce their consumption. This is the optimal response to an increase in government consumption.

Rather than partly finance the consumption shock by issuing debt, the government finds it optimal to dramatically reduce its debt level in response to the shock. Similarly to before, this response is optimal because it enables workers to significantly draw down

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19Another way that the government could potentially adjust equity levels is by altering regulatory requirements over the course of the business cycle (i.e. altering $\mu$). Berka and Zimmermann (2011), for example, analyze the impact of altering capital requirements during recessions. This policy is outside the scope of this paper. Also one can imagine that the exogenous regulatory requirement is set by depositors (not the government) in order to avoid potential losses.
Figure 3: Endogenous Policy Model Response to a One Standard Deviation Positive Government Consumption Shock

Notes: All tax rates are absolute deviations from their steady-state values and all quantities are percentage deviations from their steady-state values.
their savings while at the same time allowing the capital stock to decline only modestly. The build up of capital helps the economy to accommodate the increased demands from the government.

Similarly to before, increasing the standard deviation of shocks has a nonlinear impact on both policies and allocations. For example, consider the case where the standard deviation of government consumption shocks and the standard deviation of productivity shocks both double. This leads the standard deviation of output to increase by 103 percent, the standard deviation of the default variable $\tilde{d}(s')$ to increase by 102 percent (from 1.78 percent to 3.60 percent), and the standard deviation of total bank taxes to increase by 103 percent. Perhaps this result can partially explain why developing countries, which tend to be subject to larger economic shocks, default on their debt more frequently than developed countries.

The indeterminacy described above can also be resolved by assuming that the government lacks the ability to use capital taxation as a policy instrument (i.e. $\tau^k$ is always zero). When the indeterminacy is resolved in this way, the results are mostly similar to before. Specifically, the undetermined variables, $d$, $\tau^a$, and $\tau^B$, respond in the same direction as before to shocks. $\tau^a$ increases in response to both negative productivity shocks and positive government consumption shocks. Also as before, $d$ and $\tau^B$ decrease in response to negative productivity shocks and increase in response to positive government consumption shocks. The correlations with output are nearly identical to before. The relative standard deviation of $\tau^a$ is nearly identical to its previous value as well. However, the relative standard deviation of $d$ declines by about 45 percent and this contributes to a roughly five percent decline in the relative standard deviation of $\tau^B$.

Events from the recent European debt crisis suggest that even governments under severe financial distress are willing to spend large sums of money in order to ensure
their banking sectors are adequately capitalized. These actions coincide with the results because in the model the government optimally subsidizes the banking sector in response to negative productivity shocks.

7 Small Open Economy Model

According to IMF (2002) during the sovereign restructuring episodes in Ecuador, Pakistan, Russia, and Ukraine people attempted to move their savings out of domestic financial institutions and into foreign accounts. The governments in these countries took active measures to prevent these attempts. This pattern raises interesting questions. Specifically, are capital controls an optimal response to sovereign default induced banking crises? Also, how does optimal government policy depend on the openness of the economy?

I attempt to answer these questions by altering the model so that workers have a choice about where to put their savings. Workers can invest their money in the domestic banking sector as before, or they can invest their money in banks abroad. From here on I refer to this specification as the ‘small open economy’ version of the model. Later I will compare the results in this small open economy model to the results in the baseline (closed) model.

When workers invest abroad, they earn the risk-free world interest rate, $R$, which for simplicity is constant. The representative worker’s budget constraint now looks as follows:
\[ [1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^d(s^t)]R(s^{t-1})a^d(s^{t-1}) + [1 - \tau^f(s^{t-1})]Ra^f(s^{t-1}) = c(s^t) + a^d(s^t) + a^f(s^t) + \frac{\psi}{2}a^f(s^t)^2 \]  \hspace{1cm} (46)

where the term on the far right represents a cost associated with investing or borrowing from abroad (and is needed to close the model). In order to maintain the proper degrees of freedom in the Ramsey problem, I assume \( \tau^f(s^{t-1}) \) is set in period \( t - 1 \) (prior to the realization of shocks). The first order condition with respect to foreign deposits is the following:

\[ c(s^t) - \sigma[1 + \psi a^f(s^t)] = \beta \sum_{s^{t+1}} \pi(s^{t+1} \vert s^t)[1 - \tau^f(s^{t-1})]Ra^f(s^{t-1}) - \sigma. \]  \hspace{1cm} (47)

Now the Ramsey problem is the following:

\[
\max \sum_{i,s^t} \beta^i \pi(s^i)W(c(s^i), l(s^i), m(s^i), \Phi, \Gamma) \\
+ \Lambda(s^t)[Y(s^t) + (1 - \delta)k(s^{t-1}) + Ra^f(s^{t-1}) - c(s^t) - k(s^t) - m(s^t) - g(s^t) \\
- a^f(s^t) - \frac{\psi}{2}a^f(s^t)^2 - \gamma n(s^t)] \\
+ \chi(s^t)[\mu n(s^t) - k(s^t)] \\
- \Phi U_c^W(s_0)[[1 - \tau^a(s_0)]R_{-1}a^d_{-1} + [1 - \tau^f_{-1}]Ra^f_{-1}] \\
- \Gamma U_m^R(s_0)[[1 - \tau^k(s_0)]R^k(s_0)k_{-1} + R^k_{-1}[1 - d(s_0)]b_{-1} - R_{-1}a_{-1}],
\]  \hspace{1cm} (48)

\(^{20}\)The proof of this follows along the same lines as the proof of Proposition 2.
where \( W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) \) is defined as:

\[
W(c(s^t), l(s^t), m(s^t), \Phi, \Gamma) = \left[ U_W(c(s^t), l(s^t)) + \theta U_B(m(s^t)) \right] \\
+ \Phi \left[ U_W^l(c(s^t)) - \frac{\psi}{2} a_f(s^t)^2 \right] + U_W^l(l(s^t)) + \Gamma U_m^B(m(s^t)).
\] (49)

### 7.1 Exogenous Policy Results for Small Open Economy Model

Figure 4 shows that government default has more devastating consequences in the open-economy setting compared to the closed-economy setting. In both the closed and open-economy cases default leads to a decline in the demand for deposits and a reduction in the interest rate on deposits. In the open-economy case this leads to an increase in deposits held overseas. Because of this, in the open-economy case there is a sharper decline in the supply of domestic deposits and a sharper decline in domestic bank lending.

### 7.2 Optimal Policy Results for Small Open Economy Model

Table 3 and Figures 5 and 6 present results for when policy is set optimally and the economy is open. Interestingly, tax-related variables are more volatile when the economy is closed. In response to a negative productivity shock, tax/default rates on banks decline more if the economy is closed than if it is open. This is because when the economy is open, workers can earn interest abroad and protect against a decline in interest earnings. So, optimally there is a larger decline in domestic financial intermediation when the economy is open. This implies that when the economy is open the govern-
Figure 4: Small Open Economy Exogenous Policy Model Response to a One Percent Government Default
ment should sustain the banking sector less in response to a negative productivity shock. Also, when the economy is open, taxes have a more distortionary impact on behavior. As such, less taxation is needed in order for the government to achieve its desired impact on allocations.

Table 3: Simulated Business-Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Closed Standard Deviation</th>
<th>Open Standard Deviation</th>
<th>Closed Correlations</th>
<th>Open Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>1.30%</td>
<td></td>
<td>0.74</td>
<td>0.78</td>
</tr>
<tr>
<td>Investment</td>
<td>1.34%</td>
<td></td>
<td>0.87</td>
<td>0.72</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>2.17%</td>
<td></td>
<td>0.44</td>
<td>0.38</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.23</td>
<td></td>
<td>0.54</td>
<td>0.92</td>
</tr>
<tr>
<td>Private Sector Lending</td>
<td>0.23</td>
<td></td>
<td>0.54</td>
<td>0.92</td>
</tr>
<tr>
<td>Domestic Bank Deposits</td>
<td>1.03</td>
<td></td>
<td>0.57</td>
<td>0.72</td>
</tr>
<tr>
<td>Foreign Bank Deposits</td>
<td>0.61</td>
<td></td>
<td>-0.93</td>
<td></td>
</tr>
<tr>
<td>Government Debt</td>
<td>6.53</td>
<td></td>
<td>0.52</td>
<td>0.58</td>
</tr>
<tr>
<td>Default Tax Rate (d)</td>
<td>1.07</td>
<td></td>
<td>0.63</td>
<td>0.49</td>
</tr>
<tr>
<td>Bank Equity Tax Rate</td>
<td>0.60</td>
<td></td>
<td>0.66</td>
<td>0.43</td>
</tr>
<tr>
<td>Total Bank Tax Rate (τB)</td>
<td>1.34</td>
<td></td>
<td>0.80</td>
<td>0.62</td>
</tr>
<tr>
<td>Domestic Deposit Tax Rate</td>
<td>1.29</td>
<td></td>
<td>-0.37</td>
<td>-0.30</td>
</tr>
<tr>
<td>Foreign Deposit Tax Rate</td>
<td>0.03</td>
<td></td>
<td>-0.93</td>
<td></td>
</tr>
</tbody>
</table>

Note: All standard deviations are in percentage form except for the foreign deposit and tax-rate standard deviations.

In response to an increase in government consumption, the government increases taxes on the banking sector more when the economy is closed. This is partly because when the economy is open, workers have the option of investing abroad and the government must provide more incentives in order to finance the increase in government consumption. Also, the government raises tax rates on foreign deposits in response to
Figure 5: Small Open Economy Endogenous Policy Model Response to a One Standard Deviation Negative Productivity Shock

Notes: All tax rates are absolute deviations from their steady-state values and all quantities are percentage deviations from their steady-state values.
a negative productivity shock and reduces tax rates on foreign deposits in response to a positive government consumption shock. These tax adjustments serve to discourage fluctuations in foreign deposit levels and reduce volatility.

The results provide important insights for understanding countries’ behavior during financial and sovereign-debt crises. Specifically, the results can help explain why Ecuador and Russia froze bank deposits during their sovereign-debt crises and why Russia, Ukraine, and Pakistan imposed capital controls during their sovereign restructuring.
episodes. The results suggest this behavior is optimal because, in the model, in re-
sponse to negative shocks the government discourages capital flight by imposing taxes
on foreign deposits. These taxes help to reduce the aggregate distortions associated with
collecting tax revenue.

Also, Tomz and Wright (2007) find that there is a weak (but negative) relationship
between economic activity and sovereign default. At least for open economies, the
results support the conclusion that output and default are weakly related. For open
economies the correlation between output and \( \tilde{d} \) is slightly less than 0.5 and the standard
deviation of \( \tilde{d} \) is relatively small. As in Tomz and Wright, this suggests that default
episodes may often be driven by political factors or other considerations that are outside
the model.\(^{21}\)

8 Robustness Tests

In this section I examine the effect of altering \( \theta \), the parameter which dictates the rel-
ative weight the government puts on bankers’ utility (see Equation (21)). The results are
not very sensitive to small changes in \( \theta \). However, increasing \( \theta \) by a large amount leads
to a clear reduction in macroeconomic volatility. As described in Section 5 originally \( \theta \)
was set as follows:

\[
\theta = \frac{\bar{c} + \bar{a}}{\bar{m} + \bar{n}}.
\]

When \( \theta \) declines from its baseline value, 0.0675, to zero it has no impact on any
standard deviations.\(^{22}\) However, when the \( \theta \) increases from 0.0675 to 0.5 the absolute

\(^{21}\)See Hatchondo et al. (2009) for a model where defaults are sparked by political turnovers.
\(^{22}\)As discussed in the calibration section, changes in \( \theta \) have no impact on steady-state allocations.
standard deviation of every variable declines. For example, the standard deviation of output declines by 3.2 percent. Interestingly, the standard deviation of bank-equity taxes declines by a large 5.7 percent. These results are due to the fact that steady-state banker consumption is lower than steady-state worker utility. So, bankers’ utility function is more curved in the neighborhood of the steady state than workers’ utility function (i.e. in the model, bankers are more risk averse than workers). This implies that bankers have more to gain from a reduction in consumption volatility than workers. As bankers become more important in the government’s social welfare function, reducing volatility becomes a larger priority. However, the results indicate that very large increases in $\theta$ lead to only modest declines in macroeconomic volatility.

9 Conclusion

This paper presents a model where banks channel workers’ savings into private investments and government bonds. Banks are subject to an equity-ratio constraint and so a sovereign default or other negative shock to bank equity has harmful effects on private sector lending and output. In this context I solve for optimal tax, debt, and bond repayment policies using the primal approach. I assume that the government can commit to a set of optimal policies at the start of time.

Results indicate that optimally the government should actively adjust bond repayments and bank taxes/subsidies in response to shocks. In response to a negative productivity shock, the government optimally increases bond repayments and provides subsidies to the banking sector. This reduces the severity of banks’ equity-ratio constraint and reduces the amount that banks cut back on lending in response to the shock. Also, in response to a negative productivity shock, the government significantly increases taxes
on workers’ deposits and dramatically reduces its debt. The tax on deposits has a negative income effect on workers and this encourages workers to reduce their consumption and increase their labor supply. The decline in government debt enables the supply of deposits by workers meets the demand for deposits by banks while at the same time ensuring that a socially optimal amount is invested in the private economy. Also, given the tax increase on workers, reducing government debt helps ensure that the government’s budget constraint holds.

In response to an increase in government consumption, the government increases taxes on workers and increases taxes and/or partially defaults on banks in order to meet its increased revenue needs. The taxes efficiently provide for the increase in government consumption by, through their negative wealth effect, encouraging an increase in the labor supply and a decline in private consumption. Also, the government reduces its debt level to ensure that the needs of workers and bankers are satisfied in an optimal manner. Workers want to significantly reduce their savings because the government shock has made them worse off. However, the optimal level of private-lending declines only slightly. Reducing the government’s debt level enables bankers to continue investing in the economy at moderate levels even as workers consume large amounts of their savings. Here again, the reduction in government debt helps guarantee that the government’s budget constraint holds.

For banks, tax and default rates tend to fluctuate less when the economy is open compared to when it is closed. A key reason for this is that taxes are relatively more distortionary when savers have the option of investing their savings abroad. Also, in response to a negative productivity shock, the government taxes foreign deposits.

Overall, the results can help to explain many examples of government behavior in practice. Most importantly, the results help explain why even fiscally strained govern-
ments often are willing to spend vast sums to bail out their banking sectors. In the model, such behavior helps to stave off a significant contraction in lending.

Future research should attempt to add optimal monetary policy into this model setting and look for ways to incorporate an endogenous equity-ratio constraint on the banking sector.\(^{23}\) Also, it would be productive to see how optimal policies change when the government lacks the ability to precommit to a set of policies. Finally, it would be interesting to see how optimal policies differ when the government spends its money on valuable public goods instead of wasteful consumption and when some government bonds are held by foreigners in a model similar to the one presented. Adding these features to the model would help give more accurate quantitative results with regard to how much and under what circumstances the government should tax the banking sector and/or reduce bond repayments.

10 References

References


\(^{23}\)Dellas, Diba, and Loisel (2010) look at both optimal fiscal and optimal monetary policy but as described above their model structure is very different because the authors don’t include capital, government bonds, or distortionary taxation in their model. Also, Meh and Moran (2010) derive endogenous equity constraints but the authors don’t study optimal policy.


A Proof of Proposition 1

The purpose of this appendix is to prove Proposition 1, that the aggregate resource constraint is equal to:

$$Y(s^t) + (1 - \delta)k(s^{t-1}) = c(s^t) + m(s^t) + k(s^t) + g(s^t) + \gamma n(s^t).$$

(51)

Proof. First note that the aggregate resource constraint is the sum of the representative worker, the representative banker, and the government’s budget constraint. The representative worker’s budget constraint is the following:

$$[1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})a(s^{t-1}) = c(s^t) + a(s^t).$$

(52)
By the definition of bank equity, we know:

\[ a(s^t) = k(s^t) + b(s^t) - n(s^t). \]  

So, the worker budget constraint can be written as:

\[ [1 - \tau^l(s^t)]w(s^t)l(s^t) + [1 - \tau^a(s^t)]R(s^{t-1})[k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})] = c(s^t) + k(s^t) + b(s^t) - n(s^t). \]  

The banker budget constraint is the following:

\[ R^k(s^t)[1 - \tau^k(s^t)]k(s^{t-1}) + R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}) \]
\[ -R(s^{t-1})(k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})) + c(s^t) + k(s^t) + b(s^t) - n(s^t) = m(s^t) + [1 + \gamma + \tau^e(s^t)]n(s^t), \]

and the government’s budget constraint can be written as:

\[ \tau^l(s^t)w(s^t)l(s^t) + \tau^a(s^t)R(s^{t-1})[k(s^{t-1}) + b(s^{t-1}) - n(s^{t-1})] + R^k(s^t)\tau^k(s^t)k(s^{t-1}) + \tau^e(s^t)n(s^t) + b(s^t) = g(s^t) + \]
\[ R^b(s^{t-1})[1 - d(s^t)]b(s^{t-1}). \]

Summing the three budget constraints leads to the aggregate resource constraint:

\[ w(s^t)l(s^t) + R^k(s^t)k(s^{t-1}) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \]
\[ \gamma n(s^t). \]  

From firms’ profit maximization problem, we know:
\[ Y(s^t) + (1 - \delta)k(s^{t-1}) = w(s^t)l(s^t) + R^k(s^t)k(s^{t-1}). \]  (58)

And so the aggregate resource constraint is:

\[ Y(s^t) + (1 - \delta)k(s^{t-1}) = c(s^t) + k(s^t) + m(s^t) + g(s^t) + \gamma n(s^t). \]  (59)

\[ \square \]

B Proof of Proposition 2

Recall that Proposition 2 states the following: (i) the allocation implied by the Ramsey equilibrium maximizes (21), subject to the aggregate resource constraint in each period, Equation (18), the equity-ratio constraint in each period, Equation (9), and the following two implementability constraints (restated for convenience):

\[ \sum_{t,s'} \beta_t \pi(s') [U^W_c(s')c(s^t) + U^W_l(s')l(s^t)] = U^W_c(s^0)R_1[1 - \tau^a(s^0)]a_1 \]  (60)

and

\[ \sum_{t,s'} \beta_t \pi(s') U^B_m(s')m(s^t) = U^B_m(s^0)[R^k(s^0)[1 - \tau^k(s^0)]k_{-1} + R^b_{-1}[1 - d(s^0)]b_{-1} - R_{-1}a_{-1}]. \]  (61)

(ii) Given the fact that an allocation satisfies the above-mentioned constraints, it is
possible to construct a government policy and a price system such that the allocation, government policy, and price system satisfy the definition of a competitive equilibrium.

Proof. The proof follows along the same lines as Chari, Christiano, and Kehoe (1995). Recall that a Ramsey equilibrium maximizes the government’s social welfare function subject to two criteria: the aggregate resource constraint holds and the requirements of a competitive equilibrium are satisfied. So, to prove (i) one must show that satisfying these two criteria is equivalent to satisfying constraints (18), (9), (60), and (61). To show this, note that there are four requirements for a competitive equilibrium to be satisfied. First, the allocation has to solve the representative worker’s maximization problem. Second, the allocation has to solve the representative banker’s maximization problem. Third, wages and the return on capital must equal to the marginal products of labor and capital respectively. And fourth, the government’s budget constraint must be satisfied. Due to Walras’ Law this fourth requirement is equivalent to requiring the representative worker’s budget constraint, representative banker’s budget constraint, and aggregate resource constraint to all hold. Due to Weitzman (1973) and Ekeland and Sheinkman (1986), we know that the necessary and sufficient conditions for an allocation to solve the representative worker’s and representative banker’s maximization problems are given by these agents first-order conditions, their constraints, and their transversality conditions. So, the constraints necessary and sufficient for the four requirements of a competitive equilibrium to be satisfied are the following:

\[ w(s^t) = (1 - \alpha)z(s^t)k(s^{t-1})^\alpha l(s^t)^{-\alpha} \] (62)

and
\[ R^k(s^t) = \alpha z(s^t) k(s^{t-1})^{\alpha-1} l(s^t)^{1-\alpha} + (1 - \delta), \quad (63) \]

\[ Y(s^t) + (1 - \delta) k(s^{t-1}) = c(s^t) + m(s^t) + g(s^t) + \gamma n(s^t), \quad (64) \]

\[ \beta^t \pi(s^t) U^W_c(s^t) \leq \lambda(s^t) \quad (65) \]

with equality if \( c(s^t) > 0 \),

\[ \beta^t \pi(s^t) U^W_1(s^t) \leq -\lambda(s^t)[1 - \tau^t(s^t)] w(s^t) \quad (66) \]

with equality if \( l(s^t) > 0 \),

\[ \lambda(s^t) - \sum_{s^t+1} \lambda(s^{t+1})[1 - \tau^t(s^{t+1})] R(s^{t+1}) a(s^t) = 0, \quad (67) \]

\[ \lim_{t \to \infty} \sum_{s^t} \lambda(s^t) a(s^t) = 0, \quad (68) \]

\[ [1 - \tau^t(s^t)] w(s^t) l(s^t) + [1 - \tau^t(s^t)] R(s^{t-1}) a(s^{t-1}) = c(s^t) + a(s^t), \quad (69) \]

\[ \beta^t \pi(s^t) U^B_m(s^t) \leq \omega(s^t) \quad (70) \]

with equality if \( m(s^t) > 0 \).
\[ \sum_{s_t+1} \omega(s_t+1)[R^k(s_t+1)[1 - \tau^k(s_t^1)] - R(s_t^1)] - \chi(s_t^1)k(s_t^1) = 0, \] (71)

\[ \sum_{s_t+1} \omega(s_t+1)[1 - d(s_t+1)]R^b(s_t^1) - R(s_t^1) = 0, \] (72)

\[ [\omega(s_t^1)[1 + \gamma + \tau^e(s_t^1)] - \sum_{s_t+1} \omega(s_t+1)R(s_t^1) - \mu \chi(s_t^1)n(s_t^1) = 0, \] (73)

\[ \lim_{t \to \infty} \sum_{s_t} \lambda(s_t)b(s_t^1) = 0, \] (74)

\[ \lim_{t \to \infty} \sum_{s_t} \lambda(s_t^1)k(s_t^1) = 0, \] (75)

\[ \lim_{t \to \infty} \sum_{s_t} \lambda(s_t^1)n(s_t^1) = 0, \] (76)

\[ m(s_t^1) + [1 + \gamma + \tau^e(s_t^1)]n(s_t^1) = R^k(s_t^1)[1 - \tau^k(s_t^1)]k(s_t^1) \]
\[ + R^b(s_t^1)[1 - d(s_t^1)]b(s_t^1) - R(s_t^1)a(s_t^1), \] (77)

\[ \mu n(s_t^1) \geq k(s_t^1), \] (78)

where \( \lambda(s_t^1) \) be the Lagrange multiplier associated with the workers’ budget constraint, Equation (3), and let \( \omega(s_t^1) \) be the Lagrange multiplier associated with the bankers’ budget constraint, Equation (8).

Equations three to eight are from the representative worker’s problem. Equation (65)
is worker’s first-order condition with respect to consumption, (66) is the first-order condition with respect to labor, (67) is the first-order condition with respect to deposits, (68) is the transversality condition for deposits and the (69) is the worker’s budget constraint (restated for convenience). The next seven equations are from the bankers’ problem. Equations (70), (72), (71), (73) are the representative banker’s first-order conditions with respect to consumption, government bonds, private loans, and equity respectively. Equations (74), (75), and (76) are the transversality conditions for government bonds, private loans, and equity respectively. (77) is the representative banker’s budget constraint (restated for convenience). Finally, Equation (78) is the representative banker’s equity ratio constraint.

If an allocation satisfies (65)-(69), then it must also satisfy (60). This can be seen by multiplying (69) by $\lambda(s^t)$ and summing over all $t$ and $s^t$. Then using (67) and (68), I obtain:

$$\sum_{t,s^t} \lambda(s^t)[c(s^t) - [1 - \tau^l(s^t)]w(s^t)l(s^t)] = \lambda(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1}. \quad (79)$$

Using (65) and (66) this can be rewritten as:

$$\sum_{t,s^t} \beta^l\pi(s^t)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)] = U^W_c(s_0)R_{-1}[1 - \tau^a(s_0)]a_{-1}. \quad (80)$$

Similarly, if an allocation satisfies (70)-(86), then it also satisfies (61) and (9). This can be seen by multiplying (77) by $\omega(s^t)$ and summing over all $t$ and $s^t$. Using (71)-(73) this can be manipulated to obtain:
\[
\sum_{t,s} \left[ \omega(s^t)m(s^t) + \chi(s^t)[\mu n(s^t) - k(s^t)] \right] = \\
\omega(s^0)U_m^B(s^0)R^k(s^0)[1 - \tau^k(s^0)]k_{-1} + R_{-1}^b[1 - d(s^0)]b_{-1} - R_{-1}a_{-1}.
\]

By the Kuhn-Tucker condition, \( \chi(s^t)[\mu n(s^t) - k(s^t)] = 0 \). Then, using (70) and (86) this can be rewritten as:

\[
\sum_{t,s} \beta^t \pi(s^t)U_m^B(s^t)m(s^t) = U_m^B(s^0)R^k(s^0)[1 - \tau^k(s^0)]k_{-1} + R_{-1}^b[1 - d(s^0)]b_{-1} - R_{-1}a_{-1}.
\]

Moving on to the second part of the proof: that a price system, policy, and allocation that constitutes a competitive equilibrium can be obtained from an allocation that satisfies (18), (9), (60), and (61). First note that it is possible to recover the competitive equilibrium value of government debt, \( b(s^t) \), in each period when the variables associated with the allocation satisfy (18), (9), (60), and (61). This can be seen by multiplying (69) by \( \lambda(s^t) \) and summing across all states and periods. Using equations (65) to (69) and the fact that \( a(s^t) = k(s^t) + b(s^t) - n(s^t) \) for a specific realization of a history, \( s^r \), this is equal to:

\[
U^W_c(s^r)[k(s^r) + b(s^r) - n(s^r)] = \sum_{t=r+1}^{\infty} \sum_{s^t} \beta^{t-r} \pi(s^t|s^r)[U^W_c(s^t)c(s^t) + U^W_l(s^t)l(s^t)]
\]

or

56
\[
U_c^W(s^r)\left[k(s^r) + b(s^r) - n(s^r)\right] = \sum_{s^{r+1}} \beta \pi(s^{r+1} \mid s^r) \left[U_c^W(s^{r+1})c(s^{r+1}) + U_l^W(s^{r+1}) + U_c^W(s^{r+1})[k(s^{r+1}) + b(s^{r+1}) - n(s^{r+1})]\right].
\]

In the discrete-state space model, similarly to Chari, Christiano and Kehoe (1994), I solve for \(b(s^r + 1)\) by making use of the fact that in the government’s optimization problem \(k(s^{r+1})\) is the only endogenous state variable (this is not the case in the decentralized problem). Then Equation (84) can be written for every possible combination of \(k(s^r)\), \(g(s^r)\), and \(z(s^r)\). I solve the system of equations to find the government’s recursive decision rule for \(b(s^r + 1)\). To recover \(\tau_t\) I make use of the following condition:

\[
-\frac{U_l^W(s^t)}{U_c^W(s^t)} = [1 - \tau(s^t)]w(s^t).
\]

Equation (85) is the result of combining Equations (65) and (66). Next, to recover \(\tau^e(s^t)\) I use Equations (71)-(73), (77), and the Kuhn-Tucker condition,

\[
\chi(s^t)[\mu n(s^t) - k(s^t)] = 0 \forall t, s^t,
\]

to arrive at:

\[
\omega(s^t)[1 + \gamma + \tau^e(s^t)]n(s^t) = \sum_{s^{t+1}} \omega(s^{t+1})[m(s^{t+1}) + [1 + \gamma + \tau^e(s^t)]n(s^{t+1})],
\]

Similarly to how Equation (84) can be used to solve for \(b(s^t)\), Equation (87) can be used to solve for \(\tau^e(s^t)\).

Next, since the government maximized its social welfare function subject to the
banker’s equity-ratio constraint, Equation (78) will be satisfied in all circumstances and can be ignored when setting policies and prices. Also, as a result of the government’s maximization problem (78) will always hold with equality (i.e. $\mu n(s^t) = k(s^t)$ for all $s^t$). This is because the government’s first order condition with respect to $n(s^t)$ is the following:

$$\gamma c(s^t)^{-\sigma} = \mu \chi^G(s^t),$$

(88)

where $\chi^G(s^t)$ is the government’s Lagrange multiplier for the equity-ratio constraint. Since the left side of Equation (88) is always positive, the right side must be too. However, in order for government’s Kuhn-Tucker condition $\chi^G(s^t)[\mu n(s^t) - k(s^t)] = 0$ to hold, it must be the case that $\mu n(s^t) = k(s^t)$.

Since $\mu n(s^t)$ always equals $k(s^t)$ this implies that in the banker’s problem (86) will always hold regardless of the value of $\chi(s^t)$. This however implies there is an indeterminacy in the model: there are many possible price systems and policies that when combined with the allocation, represents a competitive equilibrium. To see this, note that so long as the Kuhn-Tucker condition $\chi(s^t) \geq 0$ is satisfied then Equations (71) and (73) constitute a set of two equations with three unknowns ($R(s^t), \tau^k(s^t)$, and $\chi(s^t)$).

There are many possible ways to resolve this indeterminacy. For example, using various forms of taxation so that $\chi(s^t)$ is always zero and requiring that capital tax rates are set one period in advance would successfully resolve the indeterminacy. When the indeterminacy is resolved in this manner, using Equation (73), the interest rate is equal to:

$$R(s^t) = \frac{\omega(s^t)[1 + \gamma + \tau^e(s^t)]}{\sum_{s^{t+1}} \omega(s^{t+1})}.$$
Equation (71) can then be used to recover $\tau^k(s^t)$ for each state $s^t$. Next, in Equation (77) replace $R^h(s^t-1)[1 - d(s^t)]$ with a new variable called $\hat{R}^h(s^t)$. Then, given the fact that all of the other policy variables have already been solved for, Equation (77) determines $\hat{R}^h(s^t)$ in each state $s^t$. It is important to remember $\tau^e(s^t)$ was set (in part) by imposing that Equation (72) must be satisfied. This ensures that Equation (72) is satisfied when setting $\hat{R}^h(s^t)$.

Given the nature of the model, $R^h(s^t-1)$ and $d(s^t)$ can be set to any values so long as $\hat{R}^h(s^t) = R^h(s^t-1)[1 - d(s^t)]$. For example, I could set $R^h(s^t-1)$ as the highest $\hat{R}^h(s^t)$ across all states in period $t$. Then $d(s^t)$ would measure the discount relative to the highest feasible payout on government bonds. One can think of the highest feasible payout on government bonds as the bonds’ face value.

Finally, given $R(s^t-1)$, Equation (69) determines $\tau^a(s^t)$ in each state. Similarly to $\tau^e(s^t)$ and Equation (72), $b(s^t)$ was set so as to guarantee that Equation (67) is satisfied in all states. Hence, I don’t need to worry about Equation (67) being satisfied when setting $R(s^t-1)$ or $\tau^a(s^t)$.

C Results from the Discrete-State Space Model

The purpose of this Appendix is to present and discuss the solution procedure, calibration, and results for the closed-economy optimal policy model in the case where the discrete state-space method is used to solve the model. The discrete state-space method allows for a universal solution, not just for a solution within a small neighborhood around the steady state. Also, assuming that the state space is large enough, a discrete-state space based solution will provide a more accurate solution than even a
second-order Taylor approximation around the non-stochastic steady state. For these reasons, solving the model using the discrete-state space method provides for a solid robustness check of the results.

C.1 Solution Procedure and Calibration for the Discrete-State Space Model

In the discrete-state space model, steady-state calculations as described in Section 4 are used to find \(\Phi\) and \(\Gamma\) as well as starting values for the capital stock (in the Ramsey model bank equity is a jumper variable and so it does not require an initial value). I assume that there are four exogenous states: \((g_h, z_h)\), \((g_h, z_1)\), \((g_l, z_h)\), and \((g_l, z_l)\), where \(g\) and \(z\) are the values for the government and technology shock respectively, and \(h\) stands for ‘high’ and \(l\) stands for ‘low’. I set \(g_h\) and \(g_l\) so that the percentage standard deviation of government consumption in the model matches the standard deviation of the detrended logarithm of government consumption in the data (annual U.S. data over the period 1947 to 2012). \(g_h\) is one percentage standard deviation above and \(g_l\) is one percentage standard deviation below the non-stochastic steady-state value of government consumption. I use values from Chari, Christiano, and Kehoe (1995) to calibrate \(z_h\) and \(z_l\), and to calibrate the probability of transitioning from one state to another. These authors assume that the probability of \(g\) transitioning to a new state is independent of the whether or not \(z\) transitions to a new state and vice versa.

The size of the capital grid is 201 points, the size of the labor grid is 101 points and the size of the banker-consumption grid is 61 points.\(^{24}\) Because the government

\(^{24}\)The distance between grid points is the following: 0.03 units between points in the capital grid (the non-stochastic steady state is 2.60), 0.01 units between points in the labor grid (the non-stochastic steady state is 0.44), and 0.0005 units between points in the banker-consumption grid (the non-stochastic steady state is 0.0068).
always forces the equity-ratio constraint to bind at equality, there is no need for a bank-equity grid. Also, it is important to note that banker consumption and worker labor are intratemporal decisions and (for the government) the only endogenous state variable in the model is capital. This implies that I can solve the model in two steps. First I solve for optimal labor input and optimal banker consumption at every possible combination of period $t$ capital, period $t+1$ capital, government consumption, and productivity level. Then, given that the social planner always makes the labor and banker-consumption choices optimally, I can iterate to convergence on the Bellman equation in order to find the optimal period $t+1$ capital stock given the period $t$ capital stock, government consumption, and productivity level. Solving the model in this manner reduces the memory demands on the computer. Using a computer with 2.0 gigabytes of RAM and an Intel Core 2 Duo processor (basically a standard personal laptop from 2008) it takes one hour and 48 minutes to solve the model in MATLAB.

Table 4: Calibration for Discrete-State Space Model

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP in high productivity state</td>
<td>$z_h$</td>
<td>1.04</td>
</tr>
<tr>
<td>TFP in low productivity state</td>
<td>$z_l$</td>
<td>0.96</td>
</tr>
<tr>
<td>High Government Consumption</td>
<td>$g_h$</td>
<td>0.23</td>
</tr>
<tr>
<td>Low Government Consumption</td>
<td>$g_l$</td>
<td>0.18</td>
</tr>
<tr>
<td>Probability of switching technology states</td>
<td></td>
<td>0.09</td>
</tr>
<tr>
<td>Probability of switching government states</td>
<td></td>
<td>0.05</td>
</tr>
</tbody>
</table>

* Refers to steady-state values.

C.2 Results for Discrete-State Space Model

Table 4 presents the results for the discrete-state space version of the model. Output is slightly more volatile than before due the new parameterization of the model. As the
results show, government debt continues to be more volatile than output and positively correlated with output. As before, this is because the government actively injects and withdraws debt from the economy to make sure that the supply of deposits by workers matches the demand for deposits by banks while ensuring that an optimal level of private investment takes place. However, now the correlation between output and government debt is much closer to zero and the volatility of hp-filtered debt has declined significantly. As before, in response to negative productivity shocks the government actively subsidizes bank equity by using both increased bond repayments and direct equity subsidies. This ensures that bank equity does not decline excessively during recessions. Total bank taxes also continue to be procyclical. However, the correlations between bank tax/default variables and output are lower than before. The government still taxes depositors in response to negative productivity shocks. This allows the government to meet its revenue requirement and impose an income effect on workers. Although the magnitude of some standard deviations and correlations have changed, overall the results suggest that the key conclusions of the model continue to hold irrespective of the solution method employed.
Table 5: Simulated Business Cycle Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\sigma(x)/\sigma(Y)$</th>
<th>$\rho(x,Y)$</th>
<th>$\rho(x,Z)$</th>
<th>$\rho(x,G)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Consumption</td>
<td>0.56</td>
<td>0.30</td>
<td>0.75</td>
<td>-0.52</td>
</tr>
<tr>
<td>Investment</td>
<td>3.25</td>
<td>0.93</td>
<td>0.83</td>
<td>0.17</td>
</tr>
<tr>
<td>Government Consumption</td>
<td>1.47</td>
<td>0.47</td>
<td>-0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>Bank Equity</td>
<td>0.25</td>
<td>0.61</td>
<td>0.52</td>
<td>0.14</td>
</tr>
<tr>
<td>Private Sector Loans</td>
<td>0.25</td>
<td>0.61</td>
<td>0.52</td>
<td>0.14</td>
</tr>
<tr>
<td>Domestic Bank Deposits</td>
<td>0.91</td>
<td>0.19</td>
<td>0.68</td>
<td>-0.64</td>
</tr>
<tr>
<td>Government Debt</td>
<td>1.59</td>
<td>0.11</td>
<td>0.64</td>
<td>-0.68</td>
</tr>
<tr>
<td>Default Tax Rate</td>
<td>4.26</td>
<td>0.14</td>
<td>0.27</td>
<td>0.08</td>
</tr>
<tr>
<td>Bank Equity Tax Rate</td>
<td>1.24</td>
<td>0.27</td>
<td>0.53</td>
<td>-0.32</td>
</tr>
<tr>
<td>Total Bank Tax Rate</td>
<td>3.25</td>
<td>0.24</td>
<td>0.43</td>
<td>0.01</td>
</tr>
<tr>
<td>Domestic Deposit Tax Rate</td>
<td>1.03</td>
<td>-0.19</td>
<td>-0.51</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Note: All standard deviations are in percentage form except for the tax-rate standard deviations.