Bilateral Accidents with Intrinsically Interdependent Costs of Precaution

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Working Paper 2002-11

September 2002
Abstract
The standard economic model of bilateral precaution postulates an interdependency between the care taken by injurers and victims that operates through the effects of each on the expected accident loss. This paper considers situations in which each party's precaution affects not only expected accident loss, but also directly affects the other party's cost of taking precaution. Generalizing the economic model of tort law in this way allows for a more complete analysis of when standard tort rules can and cannot induce optimal precaution. When this additional externality is introduced into a model of unilateral harm (where all accident losses are borne by the victim), none of the standard tort liability rules induces socially optimal behavior by both parties. Moreover, under a contributory negligence rule, the only equilibrium is in mixed strategies; this gives rise to the possibility of litigation in equilibrium. A 'tort-like' liability rule that induces socially optimal behavior by both parties is then characterized; this involves a payment by victims to non-negligent injurers whenever an accident occurs. The model is then extended to consider the case of bilateral harm (where both parties suffer accident losses). It is shown that, as long as both parties can sue to recover their accident losses, all negligence-based tort rules lead to socially optimal behavior by both parties.
1) Introduction

The standard economic model of accidents with bilateral precaution (as presented, for instance, in Shavell (1987, pp. 36f)) posits an interdependency between the levels of care taken by injurers and victims that affects the benefits from precaution (i.e. the accident losses prevented). On this basis, the model leads to the conclusion that, with legal standards of care set at the socially optimal levels, and with no litigation costs, uncertainty, misperception, error or wealth constraints, all negligence-based liability rules induce socially optimal behavior by both injurers and victims. Leong (1989) and Arlen (1990a, b; 1992) generalize the standard bilateral precaution model to accommodate bilateral harm (where accident losses are suffered by both parties). As long as each party can recover damages for its accident losses, the basic result is similar to that of the standard unilateral harm model: in the absence of litigation costs, uncertainty, misperception, error and wealth constraints, all negligence-based tort rules induce optimal behavior by both parties (Arlen, 1990a).

In the standard bilateral precaution model, each party’s cost of care depends directly only on its own level of precaution. The interdependency between the two parties’ total accident costs occurs only via the effects of care on the expected loss from the accident. This paper addresses the other side of the issue, introducing a direct interdependency in the two parties’ costs of precaution. Thus, it analyzes situations in which a change in one party’s level of care also directly influences the cost to the other party of taking any given level of care. In essence, in the standard model, parties interact only by affecting the victim’s expected benefit from precaution. In the model presented in this paper, parties also interact by directly affecting (shifting) the other party’s supply of precaution. It may be thought that this cost-side interdependency would be symmetric in relation to the standard analysis, particularly as the labeling of ‘costs’ and ‘benefits’ is somewhat arbitrary. However, we find that, when this externality is introduced into an otherwise standard unilateral harm model (where the accident losses are borne by only one party), none of the standard liability rules – no liability, strict liability, negligence, strict liability with a defense of negligence, negligence with a defense of contributory negligence, and comparative negligence (CN) – induce socially optimal care by both injurers and victims.

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1 The standard model has its origins in the analysis of Brown (1973). See also e.g. Ordover (1978) on litigation costs, Craswell and Calfee (1986) on uncertainty, and Shavell (1986) on wealth constraints.

2 The optimality result does not extend, in general, to the choice of activity level. This paper focuses on the choice of levels of care, and does not consider the issue of activity levels.
The basic intuition for this result is that these tort rules allow for accident losses to be shifted between the parties, but makes no such provision for shifting precaution costs (in particular, there is no cause of action for one party to recover its precaution costs, or part thereof, from the other). Thus, for example, under a simple negligence rule, the injurer will comply with the standard of due care. Anticipating this, the victim will choose a level of care to minimize the sum of her accident losses and her costs of precaution, but has no incentive to consider the injurer’s precaution costs (which, under our assumptions, are affected by the victim’s choice of care). It might appear that this inefficiency could be corrected simply by the court adjusting the amount of damages awarded in order to account for the cost interdependency. However, it is shown below that it is not in fact possible to do so within the limitations of traditional tort rules. The fundamental problem is that optimality requires that the legal rule forces the victim to internalize the externality affecting the injurer’s costs. In principle, it is possible to do so by modifying the damages award in a manner that depends on victim precaution; however, this will impair the incentives for the injurer to take optimal care. In order to implement the social optimum, it is necessary to consider a wider class of rules. To illustrate this point, we characterize a ‘tort-like’ mechanism that induces optimal behavior by supplementing the injurer’s accident liability with a payment by the victim to non-negligent injurers whenever an accident occurs. Of course, this rule differs significantly from existing liability rules.

A second result from this framework is that, under a contributory negligence rule, there is no equilibrium in pure strategies. Rather, the only equilibrium is in mixed strategies, which implies that the parties will choose (with some positive probability) to behave negligently in equilibrium. This gives rise to the possibility of successful litigation in equilibrium, in contrast to the standard model where parties who face negligence rules always satisfy the standard of care, so that, although there are some accidents, all parties are non-negligent. Existing answers to the question of why we have extensive, successful, tort litigation have focused on information deficiencies, error, and wealth constraints. This analysis reveals an additional possibility: in some cases, the structure of traditional tort rules cannot induce non-tortious behavior.

We also extend the model to the case of bilateral harm (where both parties may suffer accident losses). In these circumstances, as long as both parties can sue to recover their accident losses, all negligence-based tort rules lead to socially optimal behavior by both parties. Finally, the paper considers the implications of these results for the circumstances in which tort law can...
and cannot induce optimal behavior. It is argued that the results of this paper suggest that externalities can be internalized through tort liability rules if a legal standard of care and liability for damages are imposed on each party that creates externalities (regardless of how many different externalities the party generates). However, it is not necessary for the law to create as many causes of action as there are externalities. All that is needed is that there be a cause of action against each externalizing party and that courts can take account of all relevant externalities in setting legal standards. In the next section, we further explain the intuition that motivates this model, and provide some illustrative examples. The formal model for the unilateral harm case is presented in Section 3. This is then extended to the case of bilateral harm in Section 4. Section 5 discusses the wider implications of the results, and Section 6 concludes the paper.

2) Examples

Figures 1 and 2 illustrate the difference between the standard model (Shavell, 1987) and our extension of that model. In each figure, the horizontal axis represents the injurer’s level of care (x), while the vertical axis represents costs (monetized to dollars). Following the modified Hand formula, the court defines reasonable care, x*, by comparing the marginal benefits of additional precaution with the marginal costs borne by the injurer in undertaking further precaution. Marginal benefits (MB) are assumed to be constant, while marginal costs (MC) are increasing in x. For bilateral accidents, marginal benefit is a function of both x and y (where y is the victim’s level of care). In the standard model, the injurer’s marginal cost is assumed to depend only on his own precaution (i.e. MC = MC(x)), as in Figure 1. The major innovation in this paper is quite simple – it is to allow for the possibility that the injurer’s (and victim’s) marginal cost can depend on both parties’ care, as in Figure 2 where the injurer’s marginal cost, MC = MC(x; y).

[Figure 1 and Figure 2 about here]

The implications of this innovation for the ability of conventional tort rules to induce socially optimal behavior are analyzed below in Sections 3 and 4. A more concrete sense of situations described by this extended model can be gained by looking a few examples focusing on auto accidents.

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3 United States v. Carroll Towing Co., 159 F.2d 169 (2d Cir. 1947).
One way that drivers take precaution is by maintaining a safe following distance. The effort cost of doing so depends on the behavior of other drivers. The forms of precaution taken by other drivers on a multi-lane highway may take various forms: also maintaining safe following distance behind other cars, signaling lane changes, observing maximum and minimum speed restrictions, traveling at a speed consistent with the flow of traffic. Different types of precaution also involve different types of costs: delay in arriving at one’s destination, the degree of attention one has to pay to driving, the level of tension experienced in paying attention, or the opportunity cost of not being able to pay attention to the scenery or planning future activities.

Consider the following scenario. One driver takes precaution by maintaining a safe following distance; the opportunity cost is the amount of productive thought that can be given to tomorrow’s work activities. Another driver takes precaution by signaling lane changes and allowing sufficient distance between other vehicles before doing so. This is done at the cost of delay in arriving at her destination. The standard model recognizes that the level of precaution exercised by this driver in changing lanes affects the probability and perhaps severity of an accident. Our model recognizes that the level of precaution exercised by the driver changing lanes also affects the effort (cost) the other driver has to exercise in maintaining a given, safe, following distance. If the driver changes lanes with little precaution, weaving rapidly between lanes without signaling, cutting in closely in front of other cars in an effort to maintain a higher average speed he or she will also affect the other drivers’ costs of precaution. The driver trying to maintain a safe following distance has to shift more attention from planning the next days activities to driving or may experience more tension in driving. In other words, the other driver’s cost function has shifted out.

In recent years, sport utility vehicles (SUVs) have gained widespread popularity. One of the motivations for purchasing SUVs is that they are taller than most cars. As a result, the SUV driver sits higher than surrounding traffic and finds it easier to observe traffic. Consequently, it is less costly to see and anticipate other drivers’ actions (i.e. to take precaution). The standard model would recognize that the purchase of an SUV shifts in the owner’s cost of precaution and by affecting their level of precaution, affects expected accident loss. But those who continue to drive cars also find themselves behind taller, and sometimes wider, vehicles. It is now more costly for the car driver to observe and anticipate others’ actions. For example, they may be forced to move to the edge of their lane to try to see around the wider, higher vehicle. The
precautionary activity of the SUV driver has shifted the cost to the car driver of taking precaution. The standard model does not capture this impact of the purchase of SUVs on other drivers’ cost of taking care, while our model does so.

Finally, consider a less realistic scenario that nonetheless illustrates the model’s intuition particularly well. Suppose that the injurer is a driver and the victim a cyclist, and that the parties are racing, so that each party’s utility depends on its speed relative to that of the other party. Each party can take precaution to avoid an accident by reducing speed. However, even if the probability of an accident is zero, each party still cares (directly) about the other’s level of precaution (i.e. speed). Moreover, the parties’ precaution costs are directly interdependent, in the sense that, when the cyclist slows down, the driver suffers less disutility from any given reduction in her own speed. Suppose that the law requires the driver to drive no faster than the socially optimal speed in order to avoid liability, and that the expected accident liability is sufficiently large to induce the driver to satisfy this standard of care. In the standard analysis, the cyclist would then internalize all accident losses, and take socially optimal care. However, in our scenario, the cyclist’s care (i.e. reduction in speed) has two distinct effects – one is to reduce the expected accident loss, and the other is to reduce the cost to the driver of satisfying the legal standard. The cyclist will take the former fully into account, but has no incentive to consider the latter. Therefore, the cyclist will ride faster than is socially optimal, while the driver will satisfy the standard, but will have to incur a greater cost to do so than if the cyclist were behaving in a socially optimal manner.

The foregoing examples suggest scenarios in which our model may be applicable. Another source of evidence on the empirical relevance of the model is provided by the factors considered by courts in their choice of negligence standards. In particular, when justifying the imposition of a particular negligence standard, courts have on occasion explicitly referred in their opinions to the effects of a party’s care on the other party’s cost of care. For instance, in Andrews v. United Airlines, the issue was whether the airline had been negligent in failing to prevent a

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4 Emons and Sobel (1991) use a driver-cyclist example, although without the cost interdependency that we introduce.

5 Andrews v. United Airlines, 24 F-3d 39 (9th Cir. 1984). Another case that highlights the significance of cost interdependencies is Murphy v. Steeplechase Amusement Co. (166 N.E. 173 (N.Y. 1929)). The plaintiff was injured while riding “The Flopper”, an attraction at an amusement park. The salient feature of this case is that any precaution taken by the defendant (by, for instance, making the ride safer) would have directly impacted on the plaintiff’s utility (which depended in part on the risk associated with the ride). We are grateful to Warren Schwartz for suggesting these examples.
briefcase from falling from an overhead compartment and striking the plaintiff. In particular, the question was whether the airline should have installed netting in the overhead bins to reduce the probability that an object would fall. The court recognized that, even if installation of the netting would be a cost effective way to reduce expected harm, it may not be optimal because of the associated increase in the disutility experienced by passengers.

3) The Case of Unilateral Harm

3.1) The Model

The model developed in this paper follows the standard analysis of accidents between strangers with bilateral precaution, as presented in Shavell (1987, pp. 36f). All injurers (I) are assumed to be identical, as are all victims (V). As in past models, all actors are assumed to be risk neutral. Each party can take precaution that reduces the expected loss from an accident (i.e. reducing the probability of the accident, the harm if it does occur, or some combination of the two), and faces a cost of taking such precaution. In this section a unilateral harm framework is assumed; thus, if an accident occurs in spite of these precautions, the losses are borne directly only by V. Depending on the liability rule that applies, I may or may not be required to compensate V. If such a payment is made, it is assumed to perfectly compensate V for the accident loss (thus, these losses are purely pecuniary in nature).

It is assumed that the parties have complete information about their payoffs and the applicable legal rules and standards, share common prior beliefs about the probability of the accident, and are not subject to error in their choice of actions. Parties are assumed to choose their level of care so as to minimize the sum of their own costs of precaution and expected liability from accidents, given the governing tort rules. Courts are similarly assumed to have perfect information about costs, expected damages and the relationships between care and expected damages. Where tort rules involve negligence, courts are assumed to set the due care standard at the social cost-minimizing level. We will refer to this as the socially optimal or efficient level of care.

In the standard bilateral precaution model, the social objective function is to minimize the sum of expected accident losses and the costs of precaution:

\[ x + y + L(x, y) \]
where $x$ denotes I’s precaution, $y$ denotes V’s precaution, and $L$ is the expected accident loss (Shavell, 1987, p. 37). The notion of bilateral precaution is captured in the assumption that $L$ is a function of both $x$ and $y$. In this formulation, expenditures on precaution are used as the *numeraire*, so that $C^I(x) = x$, where $C^I(x)$ is I’s cost of taking precaution level $x$. Similarly, $C^V(y) = y$, where $C^V(y)$ is V’s cost of taking precaution level $y$.

Our extension of the standard model raises the question of how to appropriately represent the way in which courts set due care standards. As a general matter, a conceptual distinction can be drawn between the *level* of care taken by a party (characterized in terms of that party’s behavior) and the *cost* (whether a dollar amount or some nonmonetary cost) associated with that care. In the standard model, this distinction does not matter, as there is a one-to-one correspondence between the expenditures on care and the level of precaution. That is, the cost ($C$) of care is used as the unit by which the level ($x$) of care is measured (so that $x \equiv C(x)$). As long as a party’s costs of care depend solely on its own behavior, this simplifying assumption does not affect the results of the analysis. However, where a party’s cost of care may also depend on the other party’s behavior (i.e. where $C = C(x; y)$) the distinction between levels and costs of precaution becomes significant. As a result, the issue of whether courts define standards of care in terms of costs or levels must be faced. In practice, courts almost invariably describe standards of care in terms of the party’s behavior (e.g. the speed at which they were travelling) rather than in terms of the expenditure on care. Thus, we assume that while courts define reasonable care as a level of care such that the costs do not outweigh the benefits of taking care, the standard that an injurer faces is a standard of behavior, i.e. a level of care, $x$.

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6 For a discussion of various issues relating to what is meant by a standard of care, see Schwartz (1989).

7 For instance, in the classic case of *United States v. Carroll Towing Co.*, supra note 3, Judge Hand’s assessment of whether the barge owner was negligent focused on whether or not a bargee was present on the barge, rather than on the wages that the bargee would have had to be paid.

8 In other respects, we follow the standard model. For instance, we retain the assumption that the court sets the standard of care for each party at the socially optimal level of care for that party to take, given that the other party takes socially optimal care. A party who meets this standard of care will not be found negligent, even though the other party chooses to behave negligently. That is, the standard does not vary with the behavior of the other party.

Conceptually, a court could set a standard of care, $x^*$, that depends on the other party’s behavior, so that $x^* = x(y)$, where $x$ is the potential injurer’s care and $y$ is the potential victim’s care. In practice, courts do not do so, instead setting a standard of reasonable behavior for each party that does not change with the other party’s behavior. There is a narrow area of exception to this under the ‘last clear chance’ doctrine, where the defendant can be held liable despite the plaintiff’s negligence if the defendant could have acted to avoid the accident given the plaintiff’s negligent behavior - see *Dunn Bus Service v. McKinley* 130 Fla.778, 178 So. 865 (1937). For an economic analysis, of the ‘last clear chance’ doctrine, see Wittman (1981).
This section examines how the results of the standard model change when there are *interdependencies* between the two parties’ costs of precaution. That is, one or both of the parties’ costs of taking precaution depend not only on their own actions, but also on the other party’s action, i.e. $C^I = C^I(x; y)$ and $C^V = C^V(y; x)$. The social objective, as in Shavell (1987), is to minimize the sum of the costs of precaution and expected accident loss. However, this now takes account of the possible interdependence between the victim and injurer’s costs of precaution:

$$C^I(x; y) + C^V(y; x) + L(x, y)$$

(2)

Here, $C^I$ denotes I’s cost of precaution, and $C^V$ denotes V’s cost of precaution. The possible interdependence between them is captured by including both $x$ and $y$, which now denote the *levels* of care taken by I and V, respectively, as arguments in each cost function. It should be noted that the focus here is on the general case, where both $C^I$ and $C^V$ each depend on both $x$ and $y$. It is also possible to analyze two special cases, one where $C^I$ depends only on $x$ while $C^V$ depends on both $x$ and $y$, and the other where $C^I$ depends on both $x$ and $y$, while $C^V$ depends only on $y$. These special cases are solved separately in Dharmapala, Hoffmann and Schwartz (2001).

Following Shavell (1987, p. 36), we assume that the accident loss is non-negative and decreasing at an increasing rate in both I’s and V’s precaution levels:

**A1:** (i) $L(x, y) \geq 0$, (ii) $L_x < 0$, (iii) $L_y < 0$, (iv) $L_{xx} > 0$, (v) $L_{yy} > 0$.

It will be assumed that each actor’s cost is increasing and convex in her own level of precaution, so that:

**A2:** (i) $C^I_x > 0$, (ii) $C^V_y > 0$, (iii) $C^I_{xx} > 0$, (iv) $C^V_{yy} > 0$.

While it is possible that one party’s care may render the other’s precaution *more* costly, the focus here is on the case of positive externalities in costs of precaution. It is assumed that a higher level of care by one party lowers the other party’s cost of care:

**A3:** (i) $C^I_y \leq 0$, (ii) $C^V_x \leq 0$.

This assumption changes only the direction of deviation of equilibrium care from socially optimal care, not the basic efficiency results of this paper.

A further assumption is that the accident losses $L$ are ‘sufficiently large’ relative to the costs of precaution, in the following sense:

**A4:** (i) For any $y$, and any $x < x^*$, $L(x, y) > C^I(x^*; y) - C^I(y; x)$. 

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Thus, if courts impose a standard of care $x^*$ on $I$, it is assumed that the cost savings that $I$ can achieve by taking less care than required by the standard are always exceeded by the increase in the expected accident losses (and hence, under a negligence rule, in the expected liability). This assumption maintains the discrete jump between the injurer’s expected losses for levels of precaution below and at or above the social optimum that the standard model depends on to assure that the injurer takes optimal precaution under negligence rules (Shavell, 1987, p. 35).

It should be emphasized that this is not as restrictive an assumption as it may appear at first. Suppose that the court can impose, in addition to damages $L$ that compensate $V$ for the accident loss, a punitive penalty, represented by a nonnegative constant $D$, on $I$. Then, even if $A4(i)$ is not satisfied, it will always be possible to choose a $D$ such that $D + L$ exceeds the right hand side of the expression in $A4(i)$.

An analogous assumption is made for $V$:

$A4$: (ii) For any $x$, and any $y < y^*$, $L(x, y) > C_V(y^*; x) - C_V(y; x)$.

The assumptions $A4(i)$ and $A4(ii)$ may seem strong. However, it should be remembered that, without these assumptions, a party on which a negligence rule is imposed will not, in general, choose to satisfy that standard. The central results of this section are the nonoptimality of behavior under the standard tort rules, even when $A4$ holds. Thus, relaxing $A4$ would simply reinforce this basic result, by making nonoptimal behavior even more pervasive.\(^9\) In this sense, $A4$ is a conservative assumption, making the best possible case for the efficiency of standard tort rules.

### 3.2) Results

This section analyzes the behavior of $I$ and $V$ under six different tort liability rules: no liability (NL), strict liability (SL), simple negligence (N), strict liability with a defense of contributory negligence (SLdN), negligence with a defense of contributory negligence (NdN), and comparative negligence (CN). We follow standard definitions of these rules (see Shavell (1987, Ch. 2), and Cooter and Ulen (1997)). In the case of CN, we assume that when both parties are negligent, liability is shared; however, we do not specify a particular sharing rule.\(^{10}\) In each

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\(^9\) Additional assumptions to ensure that the SOCs for the minimization of Eq (2) are satisfied, are discussed in the Appendix.

\(^{10}\) Various sharing rules have been used with CN over time. Early admiralty cases split liability 50/50, while some variants only require sharing of liability when the victim’s actions have contributed at least 50 percent to the probability of the accident. We follow the more common case of applying comparative negligence whenever both
of the rules that involves a negligence standard, we follow the previous literature and assume that the court sets the standard of care at the socially optimal level of care, \( x = x^* \) and/or \( y = y^* \), as applicable, with no uncertainty or error.\(^{11}\)

Minimizing the social loss of accidents (Eq. (2)) with respect to \( x \) and \( y \), the FOCs are:

\[
C_x I(x, y) + C_y V(x, y) + L(x, y) = 0 \quad (3)
\]

\[
C_y I(x; y) + C_x V(y; x) + L_y(x, y) = 0 \quad (4)
\]

Assuming an interior solution, these FOCs define the socially optimal \( x^* \) and \( y^* \), given the social loss function above. The results of this section can be summarized as follows.\(^{12}\) None of the standard tort rules induce socially optimal behavior \((x^*, y^*)\) by both \( I \) and \( V \). Consider, for instance, rule N. As in the standard model, \( I \) will always satisfy the standard of due care \( x^* \), in order to avoid the discontinuous leap in accident liability that results from failing to satisfy it. Given that \( I \) satisfies \( x^* \), \( V \) will anticipate bearing all of her own accident losses; thus \( V \) will minimize the sum of expected accident losses and her own precaution costs. However, \( V \) will not take into account the precaution costs faced by \( I \) (which are affected by \( V \)’s choice of \( y \)). This leads \( V \) to take a lower level of care than is socially optimal. \( I \) will thus satisfy the standard, but will incur a higher cost in doing so than if \( V \) were behaving optimally.

This intuition can also be represented in terms of the simple diagrams in Figures 1 and 2. Suppose that the victim takes less than socially optimal care \((y' < y^*)\). In both diagrams, this shifts the MB curve downward. In the standard model, the victim’s action has no effect on the injurer’s MC function (Figure 1), but in our model the change in \( y \) also shifts the injurer’s MC function upward (Figure 2). In neither case does \( y \) affect the standard of care \( x^* \) to which the injurer is held, and, in both models, the injurer always satisfies the standard of care, \( x^* \), under a negligence rule, regardless of the victim’s action. In the standard model, should the victim choose to exercise less than socially optimal care, the injurer will still meet the due care standard, \( x^* \), and the injurer’s total cost of precaution will not change with the victim’s action. What does change is that the total social benefit (expected accident loss) resulting from both parties’ care is reduced (see Figure 1). Now consider our model, in which the victim’s care affects the injurer’s

\(^{11}\) Following the standard assumption about causality rules, it is assumed that, when a negligence standard is imposed on \( I \), then \( I \) is assumed to have caused the entire accident loss suffered by \( V \), rather than just the amount attributable to \( I \)’s negligence (see Grady (1984) and Kahan (1989) for a discussion of this issue).

\(^{12}\) A more formal statement of these results is presented in Proposition 1 in the Appendix.
costs of care (Figure 2). Suppose that the injurer continues to meet the standard of care, $x^*$. Now, should the victim decide to take less than socially optimal care, not only is total social benefit reduced, but the injurer’s total (and marginal) cost of taking socially optimal care also increases.

These results imply that, in the unilateral harm case with intrinsically interdependent costs of care, no tort rule induces socially optimal behavior. For instance, N provides I with incentives to take optimal precaution, but does not confront the V with either the external impact of her precaution on I’s cost of precaution or with the necessity of complying with a legal standard of care in order to avoid bearing the accident cost. Similarly, under SLdN, V must comply with the negligence standard to avoid bearing the accident cost. However, under SLdN, I is motivated only to minimize her own cost of precaution and the accident cost. I therefore ignores the external impact of her action on V’s cost of precaution, and fails to take socially optimal precaution. Under CN, V neither faces this external impact nor can fully avoid bearing accident cost by meeting a court-determined standard of care. As a result, V will not take socially optimal precaution and CN cannot induce socially optimal precaution from both parties.

It may appear at first that the results are simply due to our assumptions about the level of damages set by the court. In particular, it might seem that the inefficiency could be corrected simply by the court adjusting the amount of damages awarded in order to account for the cost interdependency. However, it can be shown that it is not in fact possible to do so within the limitations of traditional tort rules. Optimality requires that the legal rule forces the victim to internalize the externality imposed by her choice of $y$ on the injurer’s costs $C^I(x; y)$. In principle, it is possible to do this by setting a damages award (denoted $D$) that does not simply equal the accident loss $L(x, y)$; $D$ could, rather, be set to punish the victim for suboptimal precaution. However, this will impair the incentives for the injurer to take optimal care. To illustrate this point, suppose that (under rule N) V takes optimal care ($y = y^*$); then, I faces costs:

$$C^I(x; y^*) + D$$

Clearly, setting any $D$ other than $D = L(x, y^*)$ will lead I to choose an $x$ that differs from $x^*$; conversely, of course, if $D$ is set so that I chooses $x^*$, V will not find it privately optimal to choose $y^*$. Thus, manipulating the level of damages under rule N cannot induce socially optimal behavior by both parties. This point can be reiterated for each of the standard tort rules (in Section 3.3 below, however, we characterize a more general ‘tort-like’ mechanism that does implement the social optimum).
In the case of NdN, there is no pure strategy Nash equilibrium. It can be shown, however, that there exists a (unique) equilibrium in mixed strategies under NdN. Note first that, from the proof of the nonexistence of pure-strategy equilibria in Proposition 1 (see the Appendix), I’s best responses to any of V’s pure strategies involve taking precaution level $x$ equal to either $x^*$ or 0. Similarly, V’s best responses to any of I’s pure strategies involve taking precaution level $y$ equal to $y^*$ or $y^N$. Thus, it is possible to simplify the game induced by the NdN rule to one with a finite number of strategies (i.e. 2) for each player. The existence of an equilibrium in mixed strategies follows straightforwardly from Nash’s existence theorem.

Given the above simplification, any pair of mixed strategies can be represented by the parameters $r, q \in [0,1]$, where $r$ is the probability that I plays $x^*$ and $q$ is the probability that V plays $y^*$. It follows that the probability that I plays 0 is $(1 - r)$ and the probability that V plays $y^N$ is $(1 - q)$. Proposition 3 in the Appendix establishes the existence of a unique equilibrium in which each party randomizes over its possible pure strategies, sometimes satisfying the legal standard of due care, and at other times failing to do so. The most interesting implication of this result is the possibility of trials in equilibrium. In the standard model of liability under perfect information, the legal standard is always satisfied by the party upon whom it is imposed; thus, no party is ever negligent in equilibrium, so that litigation never occurs. Of course, accidents do occur in equilibrium, but they are never the result of negligent behavior. This also extends to the previous analysis of this paper – for instance, under N, as analyzed above, I always satisfies the standard $x^*$, so that there is no litigation in equilibrium.

However, the analysis of NdN in this paper yields substantially different implications. There is now a positive probability that, in equilibrium, one or both of the parties will fail to satisfy the standard. In some of these circumstances, trials will occur in equilibrium. In particular, consider the case where I plays $x = 0$ and V plays $y = y^*$. The equilibrium strategy specified in Proposition 2 involves I playing 0 with probability $(1 - r^0)$ and V playing $y^*$ with probability $q^0$. Thus, there is a probability $(1 - r^0)q^0$ that I will be negligent, while V satisfies the standard required to avoid contributory negligence. This situation could result in V successfully suing I to recover the accident loss, and thus involves a trial in which I is correctly found to have been negligent.
3.3) An Optimal Tort Rule

The basic result above was that, when the structure of social costs is given by Eq. (2), none of the standard tort rules induce optimal behavior by both I and V. This naturally leads to the question of whether there exists some other mechanism that can induce optimal behavior. In fact, there is a large class of mechanisms that can implement the social optimum, namely regulation. All that is needed is that the regulator requires I to choose x* and V to choose y*, and that the regulation is backed by sufficient enforcement to dissuade I and V from deviating from these levels of care.\textsuperscript{13} However, such regulatory mechanisms have heavy information requirements, generally suffer from high enforcement costs and do not provide \textit{ex post} compensation to injured parties. As a result, there are reasons why a society would want a tort rule instead of or in addition to a regulatory mechanism.\textsuperscript{14}

An important question thus remains – is there a ‘tort-like’ mechanism that can induce optimal behavior. Our goal is to find a mechanism that retains as many features of torts as possible and implements the social optimum. We will consider a mechanism to be ‘tort-like’ if it satisfies the following conditions:

\textbf{C1)} It is only triggered \textit{ex post}: i.e. transfers are made only in states of the world in which an accident has occurred. In contrast, regulation applies \textit{ex ante} to all states of the world.

\textbf{C2)} Budget balance: i.e. transfers are made only between the parties (I and V); no fines, taxes, or subsidies are imposed or provided by the government. In contrast, regulation requires an enforcement budget and a system of fines or other punishments, so that regulatory mechanisms may run either a budget surplus or deficit.

In order to derive a tort-like rule that implements the social optimum, it is necessary to reexamine and ‘decompose’ the accident loss function \(L(x, y)\). Recall that \(L\) captures both the probability and the severity of an accident; thus, it can also be written as follows:\textsuperscript{15}

\begin{equation}
L(x, y) = p(x, y)H(x, y)
\end{equation}

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\textsuperscript{13} It is worth pointing out that imposing strict liability on \textit{both} parties, which generally induces optimal behavior in the standard model (in the absence of collusive behavior) will not do so in our model. The parties will take suboptimal care because of the cost externality, even when each faces the full losses from accidents. This is sometimes referred to as ‘Earl Thompson’s liability rule’ – see Hindley and Bishop (1983).

\textsuperscript{14} For analyses of the relationship between regulation and tort law, see, for instance, Shavell (1984a, b), Kolstad, Ulen and Johnson (1990) and Schmitz (2000).

\textsuperscript{15} This distinction was not relevant to the preceding analysis; thus, it was convenient to simply work with \(L(x, y)\).
where $p$ is the probability of an accident and $H$ is the harm that results if an accident occurs (note that each of these depends on both $x$ and $y$).

Now consider a rule that we will call negligence with compensation for increased costs of precaution (NCC). This is a simple negligence rule augmented by the payment by V to I of compensation for I’s increased costs of precaution. The rule can be specified as follows:

NCC: If an accident occurs, and I is negligent (i.e. $x < x^*$), then I pays damages $L(x, y)$ to V. If I is non-negligent (i.e. $x \geq x^*$), then V bears her own accident losses, and also pays I an amount $\left[C_I(x^*, y)/p(x^*, y)\right]$.

Given the assumption of risk-neutrality, NCC leads to socially optimal behavior by both I and V. A formal statement and proof of this result are provided in the Appendix. The basic intuition is as follows. When I takes care $x$ and V takes care $y$, the probability of an accident is $p(x, y)$. If I chooses $x^*$, she receives a payoff of 0, regardless of V’s choice of $y$ (note that the probability of being involved in an accident $p(x^*, y)$ and the denominator of the transfer from V if there is an accident cancel out). In expectation, I bears zero precaution costs and bears no accident liability. However, if I were to take suboptimal precaution, she would face both her own precaution costs and the expected accident losses. Thus, I will always satisfy the standard of due care. Given this, V faces not only her own precaution costs and the accident losses, but also bears the expected value of I’s precaution costs (through the transfers made when accidents occur). Thus, V’s decision problem is aligned with the social planner’s, so that V chooses the socially optimal level of precaution $y^*$.

Clearly, although it is ‘tort-like’ in the sense that we have defined, NCC bears little resemblance to real-world liability rules and is an unlikely candidate for actual adoption by

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16 It should be noted that the payment from V to I is not because I has a cause of action against V for bringing a frivolous suit. The compensation V pays to I is for increased costs I bears in taking precaution to prevent V’s injury, i.e. in meeting the legal standard of care, not for the costs imposed by a baseless suit. A ‘frivolous suit’ is “litigation . . . instituted solely with the intent of harming the other party without either excuse or justification. ... Under some authority, the existing torts of malicious abuse and malicious use of process have been re-defined as a single cause of action for abusive litigation. Under the common-law claim, any party who asserts a claim, defense or other position with respect to which there exists such a complete absence of any justiciable issue of law or fact that it could not reasonably be believed that a court would accept it, or any party who brings and defends an action... that lacks substantial justification ... shall be liable in tort to the opposing party who suffers damage thereby.” (86 C.J.S. Torts Sec. 86.7 (1997)).
courts. This simply underlines the point that it requires a very unusual mechanism to induce socially optimal behavior by both parties under these circumstances.

4) The Case of Bilateral Harm

In real-world accidents, it is often the case that harm is suffered by both parties, rather than only by V. Thus, Leong (1989) and Arlen (1990a, b; 1992) generalize the standard bilateral precaution model to accommodate bilateral harm. They find that, as long as each party can recover damages for its accident losses, and in the absence of litigation costs, uncertainty, misperception, error and wealth constraints, all negligence-based tort rules induce optimal behavior by both parties (Arlen, 1990a). This section of the paper extends the unilateral harm model of Section 3 to the case of bilateral harm. Thus, the model in this section involves both bilateral precaution and bilateral cost externalities (as in Section 3) as well as bilateral harm. To facilitate comparisons with our earlier results, the notation involving I and V will be retained, although, of course, these terms are less meaningful when both parties suffer harm. The standard tort rules will be assumed to generalize straightforwardly to the bilateral harm context (as in Arlen (1990a)); thus, under a negligence rule (N) I can sue V to recover her accident losses if V is negligent, and V can sue I when I is negligent.

The social cost of accidents in the bilateral harm context can be represented as:

$$C_I(x; y) + C_V(y; x) + L_I(x, y) + L_V(x, y)$$

where $L_V(x, y)$ is the expected accident loss faced by V, and $L_I(x, y)$ is the expected accident loss faced by I. The set of assumptions required is closely analogous to that in Section 3, and these are specified formally in the Appendix (Assumptions A5-A9). These assumptions require that the accident losses $L_I$ and $L_V$ are ‘sufficiently large’ relative to the costs of precaution (or, more specifically, to the precaution costs that a party can avoid by failing to take care). Thus, they are straightforward generalizations of those in Section 3.

The court imposes social cost minimizing standards of care, $x^*(y^*)$ and $y^*(x^*)$, on both parties. The basic result (stated formally in Proposition 4 in the Appendix) is that all of the standard negligence-based tort rules – N, SLdN, NdN and CN – induce each party to take this socially optimal level of care. The intuition can be clarified by considering the basic negligence rule, N. Each party faces a choice between satisfying the standard of due care ($x^*$ for I and $y^*$ for V) and thereby avoiding all liability for the other party’s accident losses, or failing to satisfy the
standard and bearing all of the other party’s accident losses. Provided that the other party’s losses are sufficiently large, relative to the savings in precaution costs by failing to take care, each party will find it in her interest to satisfy the standard, regardless of the behavior of the other party. There is thus a unique equilibrium in dominant strategies, \((x^*, y^*)\); moreover, this coincides with the social optimum. Analogous (though somewhat more complicated) reasoning establishes that each of the other negligence-based rules also induces socially optimal behavior.

It is important to note that the intrinsic interdependency between the costs of precaution of the two parties does not lead to suboptimal behavior in the bilateral harm case. This is because each party faces a standard of due care, which, by assumption, is defined by the court so as to take into account the cost externality. A party that fails to satisfy this standard will face a discontinuous jump in its expected costs, as it will be forced to bear the accident losses of the other party. This results in the cost externality being internalized by each party (along with the accident externality). The lesson of this section is thus that, in the bilateral harm case (in contrast to the unilateral harm case) traditional tort rules are robust to the introduction of cost externalities.

5) Discussion

Having analyzed both the unilateral and bilateral harm cases, it remains to review the paper’s results and to draw some more general conclusions concerning the design of tort liability rules. The results from sections 3 and 4 are summarized in Table 1, along with results from the existing literature.

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The first two of these are the results of the standard models with independent costs of precaution, for the unilateral harm and bilateral harm cases (as discussed, for instance, in Shavell (1987) and Arlen (1990a), respectively). The next two, which are explicitly derived in Dharmapala et al. (2001), are special cases of the results in Proposition 1 (Section 3).

Model 3 considers the case where, within a unilateral harm framework, V’s precaution costs are affected by I’s level of care, but where I’s cost of care is independent of V’s precaution. In these circumstances, courts can set a negligence standard for I that takes into account the external effects of I’s precaution on V’s costs. Given that the accident loss is sufficiently large, I will always choose to adhere to this standard, as failing to do so entails a large discontinuous
jump in expected liability. Given that I takes the socially optimal level of care, V’s problem (of choosing y to maximize $C^V(y; x^*) + L^V(x^*, y)$) is identical to the social planner’s program in choosing y. Thus, all of the negligence-based tort rules lead to optimal behavior by both I and V.

The fourth model is also one of unilateral harm, and involves the case where I’s precaution costs are affected by V’s level of care, but where V’s cost of care is independent of I’s precaution. Once again, I will adhere to the standard of care imposed by the court. Anticipating this, V knows that she will bear her accident losses and her own costs of precaution; however, she does not bear I’s precaution costs, although these depend in part on her actions, and so will not choose the socially optimal level of care (this is essentially the same intuition as that in Section 3). Thus, the standard tort rules do not induce both parties to behave optimally (although NCC, the rule constructed in Section 3, does so).

Table 1 summarizes these results, along with those of this paper. Model 5 is simply the unilateral harm framework analyzed in Section 3, while Model 6 is the bilateral harm model from Section 4. For each of these models, the table shows which tort rules lead to socially optimal behavior by both I and V. Note that, in addition to the standard rules considered in the literature, the table includes NCC, the hypothetical rule derived in Section 3. This is purely for purposes of comparison, and is not intended to suggest that NCC is necessarily an appropriate rule to adopt in practice. While NCC induces optimal behavior under all the circumstances considered, it should be noted that, for Models 1-3, it involves an arbitrary wealth redistribution from victims to injurers, without having any behavioral effects.

Consider the first two rows of Table 1. In Model 1, there is only one externality (I’s effect on V’s accident loss); this externality can be internalized by imposing a negligence standard on I, and enabling V to sue I to recover her accident losses when this standard is not met. Model 2 extends this idea to the case of two externalities (I’s effect on V’s accident loss, and V’s effect on I’s accident loss). Here, the externalities can be internalized by imposing a negligence standard on each party, and enabling each to sue the other to recover accident losses when the standard is not met. Thus, it would seem from the existing literature that tort liability rules succeed in internalizing externalities by creating a cause of action for each external effect (in contrast, Leong (1989) is a model in which there are two externalities and only one cause of action, which leads to nonoptimal behavior).
The results of this paper, however, tend to cast some doubt on this generalization. Note, for example, that Model 6 (from Section 4) involves four externalities – each party’s effect on the other’s accident loss, and each party’s effect on the other’s precaution costs. However, liability rules that involve only two causes of action – i.e. which impose a negligence standard on each party, and enable the other to sue for accident losses (but not precaution costs) when the standard is violated – are sufficient to internalize all four externalities. Similarly, in Model 3, there are two externalities (I’s effect on $C^V$ and her effect on $L^V$), but they can both be internalized by imposing a negligence standard on I, and enabling V to sue to recover $L^V$.

One approach to drawing out the implications of these results is the following. Examining the totality of Table 1, it appears that tort liability rules can internalize externalities when

(i) a negligence standard that takes into account all the externalities created by a party, and
(ii) liability for damages when the standard is violated

are imposed on each party that generates externalities (regardless of how many externalities that party creates). Thus, it is not necessarily required that there be as many causes of action as externalities.

Moreover, comparison of the results of in the unilateral and bilateral harm case with parties affecting each others’ costs of precaution reveals something fundamental about the way in which torts functions to induce optimal precaution. The critical features of a socially optimal tort system are that there is a cause of action against each externalizing party, and that in setting that party’s standard of care the court takes into account all external impacts including impacts on injury and on cost of precaution. Negligence rules governing torts involving bilateral harm can induce socially optimal behavior even in the presence of external effects on the costs of precaution because there is always a cause of action available against the externalizing party. This is not so in the case of torts involving unilateral harm.

This formulation explains the results of the previous literature (Models 1 and 2). Note in particular that in Leong (1989), there are two parties – I and V – generating externalities, but a negligence standard and liability for damages are imposed only on I; thus, there is an externality-generating party (V) who does not face liability. Essentially similar explanations can be given for why Models 4 and 5 lead to suboptimal behavior: V imposes an externality on I’s precaution costs, but faces no liability for damages.
At first glance, this formulation may seem to run counter to the basic intuition from public finance that one policy instrument is needed to achieve each policy objective. Here the objectives appear to be internalizing each party’s loss from accidents and internalizing each party’s impact on the other’s cost of precaution. In this case, one might think that four policy instruments are needed. However, the comparison of results from the unilateral and bilateral harm cases show that the objective is actually to induce each party to internalize the costs (whether in terms of harm or in terms of increased cost of precaution) that they impose on the other party. Thus, with the objectives properly defined, it is apparent that only two instruments are needed. The two causes of action involved in torts with bilateral harm provide the two needed instruments. Similarly, the optimal mechanism derived above in the unilateral harm case also makes use of two policy instruments. It should be emphasized that both elements highlighted above – the negligence standard and the liability for damages – are important in the internalization of externalities. For instance, consider Model 5 (from Section 3). Rule NdN (contributory negligence) imposes negligence standards on both I and V, but a cause of action exists only for V. That is, only V is able to recover damages. Thus, as V creates an externality, but does not face liability for damages, NdN is incapable of inducing both parties to behave in a socially optimal manner.

6) Conclusion

In conclusion, this paper has analyzed the consequences that ensue when parties in accidents have intrinsically interdependent costs of precaution. This generalization of the standard economic analysis of tort rules enables a better understanding of the conditions under which torts rules can induce socially optimal precaution. The results show conditions under which standard tort rules can fail to induce socially optimal behavior, and result in successful tort litigation (even where there is no error, misperception, incomplete information or wealth constraints). We develop a tort-like rule that will induce socially optimal precaution under even these conditions. While this rule is unlikely to be functional, it does help illustrate the elements necessary for a tort system to induce socially optimal precaution. The larger contribution of this analysis is to identify those elements. This analysis shows that in order induce socially optimal behavior, all that is required of a tort system is that a cause of action be available against every externalizing party and that the cause of action involve a standard of care that accounts for all
external costs caused by that party. This is exactly what negligence rules do in most cases that would come before a court.

References


Appendix

Second Order Conditions:

The following establishes a sufficient set of conditions for the SOCs for the social problem in the general case (Eq. 2). The SOCs for the various other programs considered in the paper will be satisfied under very similar circumstances.

Let \( f \equiv C^I(x; y) + C^V(y; x) + L(x, y) \)

The SOCs require that the Hessian is positive semidefinite (i.e. the principal minor determinants are all positive). The first principal minor is

\[ f_{xx} = C'^I_{xx}(x, y) + C'^V_{xx}(x, y) + L_{xx}(x, y) \]

Note that, by assumption, \( C'^I_{xx}(x, y) > 0 \) and \( L_{xx}(x, y) > 0 \); thus, imposing the restriction that \( C'^V_{xx}(x, y) > 0 \) is sufficient to ensure that \( f_{xx} > 0 \).
The second principal minor determinant is $f_{xx}f_{yy} - f_{xy}f_{yx}$; assuming that $C^I_{xy}(x, y) > 0$ is sufficient to ensure that $f_{yy} > 0$. In addition, assuming that the cross-partial $C^I_{xy}(x, y)$, $C^I_{yx}(x, y)$, $C^V_{xy}(x, y)$, $L_{xy}(x, y)$, and $L_{yx}(x, y)$ are all sufficiently small ensures that $f_{xx}f_{yy} - f_{xy}f_{yx} > 0$. Under those conditions, the SOCs for Eq. 2 are satisfied.

**Proposition 1:** Suppose that A1-A4 holds. Then,

(i) the social optimum $(x^*, y^*)$ is not a Nash equilibrium under any liability rule

(ii) the unique (suboptimal) equilibrium under $N$ is $(x^*, y^N)$, where $y^N \equiv \arg\min C^I(y; x^*) + L(x^*, y) < y^*$

(iii) the unique (nonoptimal) equilibrium under $NL$ is $(0, y^{NL})$, where $y^{NL} \equiv \arg\min C^V(y; 0) + L(0, y)$

(iv) the unique (nonoptimal) equilibrium under $SL$ is $(x^{SL}, 0)$, where $x^{SL} \equiv \arg\min C^I(x; 0) + L(x, 0)$

(v) the unique (suboptimal) equilibrium under $SLdN$ is $(x^S, y^*)$, where $x^S \equiv \arg\min C^I(x; y^*) + L(x, y^*) < x^*$

(vi) $(x^*, y^*)$ is not an equilibrium under $CN$; there will exist suboptimal equilibria $(x^{CN}, y^{CN}) \neq (x^*, y^*)$ under $CN$ if there exist $x^{CN}$ and $y^{CN}$ that (simultaneously) satisfy the following conditions:

$C^I(x^{CN}, y^{CN}) + \alpha(x^{CN}, y^{CN})L(x^{CN}, y^{CN}) \leq C^I(x; y^{CN}) + \alpha(x, y^{CN})L(x, y^{CN}) \quad \forall x \neq x^{CN}$ (va)

$C^V(x^{CN}, y^{CN}) + (1 - \alpha(x^{CN}, y^{CN}))L(x^{CN}, y^{CN}) \leq C^V(x^{CN}, y) + (1 - \alpha(x^{CN}, y))L(x^{CN}, y) \quad \forall y \neq y^{CN}$ (vb)

where $\alpha(x, y) \in [0, 1]$ is the fraction of liability borne by $I$ when $I$ takes precaution $x \leq x^*$ and $V$ takes precaution $y \leq y^*$.\(^{17}\)

(vii) there is no pure-strategy Nash equilibrium under $NdN$.

**Proof of Proposition 1:**

(i) It will be apparent from the reasoning below $(x^*, y^*)$ is not an equilibrium under any of the rules considered here.

(ii) Consider $N$: by satisfying the legal standard of care $x^*$, I avoids all liability, and faces cost $C^I(x^*; y)$, while taking $x < x^*$ leads to costs $C^I(x; y) + L(x, y)$. Given A4(i), $L(x, y) > C^I(x^*; y)$ -

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\(^{17}\) This is the most general formulation. In fact, CN entails that $\alpha(x^*, y) = 0$. 22
C^I(x; y), I’s dominant strategy (for any y) will be to satisfy the standard. Given that I satisfies x^*, V faces C^V(y; x^*) + L(x^*, y); the FOC for V’s minimization problem differs from the FOC for the social planner’s problem in Eq (14). V will take precaution of y^N. Thus, (x^*, y^N) is an equilibrium; uniqueness follows as x^* is a dominant strategy, and y^N is the unique maximizer of V’s program.

To show that y^N < y^*, recall the FOC for the socially optimal choice of y, y^*, above. As C^I_y(\cdot) < 0 by assumption, it follows that:

\[ C^V_\gamma(y^*; x^*) + L_\gamma(x^*, y^*) > C^V_\gamma(y^N; x^*) + L_\gamma(x^*, y^N) \]

By assumption, C^V_y(\cdot) > 0 and L_\gamma(\cdot) > 0. Thus, both the LHS and RHS of the expression above represent an increasing function of y, say, f(y), where f(y) ≡ C^V_\gamma(y; x^*) + L_\gamma(x^*, y) and f'(y) > 0. It follows that, as f(y^*) > f(y^N), it must be true that y^N < y^*.

(iii) Under NL, I never faces liability for V’s loss, so her costs are only her cost of precaution, C^I(x; y), which is minimized by taking x = 0. Since C^I_x < 0, this is true for all y. Given I’s choice of x = 0, V faces cost C^V(y; 0) + L(0, y), which is minimized by y^NL. Since the FOC for this optimization problem, C^V_y(y; 0) + L_\gamma(0, y) = 0, differs from that of the social planner (eq. 11), the victim will not choose the socially optimal level of precaution, y^NL ≠ y^*. The outcome (0, y^NL) will be a unique equilibrium, but will not be socially optimal.

(iv) Under SL, V is always compensated and bears only his cost of precaution, C^V(y; x), regardless of I’s precaution. V minimizes C^V(y; x) by taking no precaution, y = 0. In turn, faces costs, C^I(x; 0) + L(x, 0), which he will minimize by taking precaution x^SL. The FOC for this minimization problem, C^I_x(x; 0) + L_\gamma(x, 0) = 0, differs from that for the social problem (Eq (10)). As a result, I’s precaution, x^SL, will differ from the socially optimal precaution, x^*. The outcome (x^SL, 0) will be a unique equilibrium, but will not be socially optimal.

(v) Under SLdN, regardless of I’s action, V can avoid bearing the expected accident loss only by taking the socially optimal level of care, y^*. V can either avoid liability by meeting the social standard of care, y^*, and face only the cost of precaution, C^V(y^*; x), or can take care y<y^* and bear both the cost of precaution and the accident loss, C^V(y; x) + L(x, y). As long as A4(ii) is satisfied, V will meet the social standard of care, y^*. Given that V takes precaution y^*, I will face and choose x to minimize both costs of precaution and accident loss, C^I(x; y^*) + L(x, y^*) by choice of x^S. The FOC for this minimization problem differs from that of the social planner’s problem in Eq 13.

To show that x^S < x^*: As C^I_x(\cdot) < 0 by assumption, it follows that:

\[ C^I_x(x^*; y^*) + L_\gamma(x^*, y^*) > C^I_x(x^S; y^*) + L_\gamma(x^S, y^*) \]

and therefore (as C^I_x(\cdot) > 0 and L_\gamma(\cdot) > 0) that x^S < x^*.

(vi) Consider CN: To show that (x^*, y^*) is not an equilibrium, suppose that I plays x^*. V faces C^V(y; x^*) + L(x^*, y) and takes precaution y^N. Therefore, (x^*, y^*) is not an equilibrium. To show that (x^CN, y^CN) ≠ (x^*, y^*) may be an equilibrium: suppose that Condition (va) holds. Then, x^CN is I’s best response to V playing y^CN. Suppose that Condition (vb) holds. Then, y^CN is V’s best response to I playing x^CN. Thus, if 3(x^CN, y^CN) such that Conditions (va) and (vb) are simultaneously satisfied, then (x^CN, y^CN) is an equilibrium. Furthermore, note that if (x^CN, y^CN) is an equilibrium, then (x^CN, y^CN) ≠ (x^*, y^*), as (x^*, y^*) is not an equilibrium.
(vii) Consider NdN:
- Suppose $x = x^*$: $V$ faces $C^V(y; x^*) + L(x^*, y)$, and thus takes precaution $y^N < y^*$. Thus, $(x^*, y^*)$ is not an equilibrium. Moreover, if $V$ takes any level of care below $y^*$, I will face no liability, and will thus take no care ($x = 0$). Thus, there cannot be an equilibrium in which I takes $x^*$ and $V$ takes $y < y^*$.
- Suppose $x < x^*$: If A4(ii) holds, $V$ will take $y^*$. Thus, there cannot be an equilibrium in which both parties take suboptimal care. Moreover, if $V$ takes $y^*$, I (given A4(i)) will take $x^*$; thus, there cannot be an equilibrium in which I takes $x < x^*$ and $V$ takes $y^*$.
This exhausts all the possibilities, so there is no equilibrium in pure strategies.

**Proposition 2:** Suppose that A1-A4 hold. Then, there exists a unique Nash equilibrium $(r^0, q^0)$ under NdN, where:

$$
 r^0 = \frac{C^V(y^*; 0) - C^V(y^N; 0) - L(0, y^N)}{C^V (y^N; x^*) + L(x^*, y^N) - C^V (y^*; 0) - C^V (y^N; 0) - L(0, y^N)}
$$

and

$$
 q^0 = \frac{C^I(x^*; y^N) - C^I(0; y^N)}{C^I (x^*, y^N) + C^I (0, y^N) + L(x^*, y^N) - C^I (x^*, y^N) - C^I (0; y^N)}
$$

**Proof of Proposition 2:**

Consider I’s expected payoff from playing $r$, given that $V$ plays $q$:

$$
r [r q C^I(x^*; y^N) - (1 - r) C^I(x^*; y^N) - (1 - r)(1 - q) C^I(0; y^N) - (1 - r) q [C^I(0; y^*) + L(0, y^*)] - (1 - q) C^I(0; y^N) + L(0, y^N)]
$$

Simplifying, the expected payoff is

$$
r [- q C^I(x^*; y^N) - C^I(x^*; y^N) + q C^I(0; y^N) + q L(0, y^N) + C^I(0; y^N) - q C^I(0; y^N)] - q [C^I(0; y^N) + L(0, y^N)] - (1 - q) C^I(0; y^N)
$$

Setting the coefficient of $r$ in the above expression equal to zero, and rearranging, yields $q^0$ (note that, using assumptions A1-A4, it follows that $q^0 \in (0,1)$). If $q > q^0$, then I’s payoff is increasing in $r$, so that I’s best response $r^*(q) = 1$ (i.e. playing the pure strategy $x^*$). If $q < q^0$, then I’s payoff is decreasing in $r$, so that $r^*(q) = 0$ (i.e. playing the pure strategy $0$). If $q = q^0$, then I’s payoff is constant in $r$, so that any $r$ is a best response to $V$ playing $q = q^0$.

Now consider $V$’s expected payoff from playing $q$, given that I plays $r$:

$$
q [- q C^V(y^*; x^*) - L(0, y^N)] - C^V(y^N; 0) + r C^V(y^*; 0) + r [C^V(y^N; x^*) + L(x^*, y^N)] + C^V(y^N; 0) + L(0, y^N) - r [C^V(y^N; 0) + L(0, y^N)] - q C^V(y^N; 0) - C^V(y^N; 0) - L(0, y^N)
$$

Setting the coefficient of $q$ in the above expression equal to zero, and rearranging, yields $r^0$ (note that, using assumptions A1-A4, it follows that $r^0 \in (0,1)$). If $r > r^0$, then $V$’s payoff is decreasing in $q$, so that $V$’s best response $q^*(r) = 0$ (i.e. playing the pure strategy $y^N$). If $r < r^0$, then $V$’s payoff is increasing in $q$, so that $q^*(r) = 1$ (i.e. playing the pure strategy $y^*$). If $r = r^0$, then $V$’s payoff is constant in $q$, so that any $q$ is a best response to $I$ playing $r = r^0$.

In particular, $q^0$ is a best response by $V$ when I plays $r^0$; moreover, from above, $r^0$ is a best response by I when $V$ plays $q^0$. Thus, $(r^0, q^0)$ is a Nash equilibrium.

To show uniqueness, suppose that there exists an equilibrium $(r', q')$, where $q' \neq q^0$.
If \( q' > q^0 \), then \( r^*(q') = 1 \). Moreover, \( q^*(1) = 0 \), so that \((r', q') = (1, 0)\). But, this cannot be an equilibrium (see proof of Proposition 5.3).

If \( q' < q^0 \), then \( r^*(q') = 0 \). Moreover, \( q^*(0) = 1 \), so that \((r', q') = (0, 1)\). But, this cannot be an equilibrium (see proof of Proposition 5.3).

Similar reasoning holds for \( r' \neq r^0 \). Thus, \((r^0, q^0)\) is the unique Nash equilibrium.

**Proposition 3:** Given A1-A4 and the risk-neutrality of \( I \) and \( V \), the unique Nash equilibrium outcome under NCC is \((x^*, y^*)\)

**Proof of Proposition 3:**

I’s payoff from satisfying \( x^* \) (for any choice \( y \) by \( V \)) is:
\[
[p(x^*, y)/p(x^*, y)]C^I(x^*; y) - C^I(x^*; y)
= 0 \text{ (for any } y) \]
while I’s payoff from failing to satisfy \( x^* \) is:
\[
-C^I(x; y) - L(x^*; y)
< 0
\]
Thus, I always satisfies \( x^* \). Given this, \( V \) faces costs
\[
[p(x^*, y)/p(x^*, y)]C^I(x^*; y) + C^V(y; x^*) + L(x^*, y)
= C^I(x^*; y) + C^V(y; x^*) + L(x^*, y)
\]
But, this is identical to the social planner’s problem in choosing \( y \). Thus, \( V \) will choose \( y^*(x^*) \), which is unique by assumption. It follows that \((x^*, y^*)\) is the unique Nash equilibrium outcome.

**Assumptions A5-A9:**

**A5:**

(i) \( L^V(x, y^*) > [C^I(x^*; y^*) + L^I(x^*, y^*)] - [C^I(x; y^*) + L^I(x^*, y^*)] \quad \forall x < x^* \)

(ii) \( L^I(x^*, y) > [C^V(x^*; y^*) + L^V(x^*, y^*)] - [C^V(x^*; y) + L^V(x^*, y)] \quad \forall y < y^* \)

**A6:**

(i) \( L^V(x, y) > [C^I(x^*; y) - C^I(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

(ii) \( L^I(x, y) > [C^V(x; y^* - C^V(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

**A7:**

(i) \( L^V(x, y^*) > [C^I(x^*; y^*) + L^I(x^*, y^*)] - [C^I(x; y^*) + L^I(x^*, y^*)] \quad \forall x < x^* \)

(ii) \( L^I(x^*, y) > [C^V(x^*; y^*) + L^V(x^*, y^*)] - [C^V(x^*; y) + L^V(x^*, y)] \quad \forall y < y^* \)

**A8:**

(i) \( L^V(x, y^*) > [C^I(x^*; y) - C^I(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

(ii) \( L^I(x, y) > [C^V(x; y^*) - C^V(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

It is assumed that under CN, if an accident occurs and both parties are deemed to be negligible, then I bears a fraction \( \alpha \in [0, 1] \) of the total accident losses, while \( V \) bears \((1 - \alpha)\). Then, the following assumptions are required:

**A9:**

(i) \( \alpha[L^I(x, y) + L^V(x, y)] > [C^I(x^*; y) - C^I(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

(ii) \( (1 - \alpha)[L^I(x, y) + L^V(x, y)] > [C^V(x; y^*) - C^V(x; y)] \quad \forall x < x^* \) and \( \forall y < y^* \)

**Proposition 4:** In the bilateral harm model with social costs given by Eq. (6), and given A5-A9, the unique Nash equilibrium outcome under all negligence-based liability rules (N, SLdN, NdN and CN) is \((x^*, y^*)\).

**Proof of Proposition 4:**

N: Consider I’s problem, assuming that \( V \) chooses \( y^* \). Then, I faces expected costs
\[
C^I(x^*; y^*) + L^I(x^*, y^*)
\]
by choosing \( x = x^* \), and
\[
C^I(x; y^*) + L^I(x, y^*) + L^V(x, y^*)
\]
By choosing \( x < x^* \), given \( A5(i) \), I will choose \( x = x^* \). Now suppose that V chooses \( y < y^* \); then, I faces costs \( C^I(x^*; y) \) by choosing \( x = x^* \), and \( C^I(x; y) + L^V(x, y) \) by choosing \( x < x^* \). Given \( A6(i) \), I will choose \( x = x^* \). Thus, \( x = x^* \) is a dominant strategy for I. By the symmetry of the problem, and given \( A5(ii) \) and \( A6(ii) \), V will choose \( y = y^* \). Hence, the unique equilibrium outcome is \((x^*, y^*)\).

SLdN: Consider I’s problem, assuming that V chooses \( y = y^* \). Then, I faces expected costs
\[
C^I(x^*; y^*) + L^V(x^*, y^*)
\]
by choosing \( x^* \), and
\[
C^I(x; y^*) + L^I(x, y^*) + L^V(x, y^*)
\]
by choosing \( x < x^* \). Given \( A7(i) \), I will choose \( x^* \). Now suppose that V chooses \( y < y^* \); then, I faces costs \( C^I(x^*; y) \) by choosing \( x = x^* \), and \( C^I(x; y) + L^I(x, y) \) by choosing \( x < x^* \). Given \( A8(i) \), I will choose \( x = x^* \). Thus, \( x = x^* \) is a dominant strategy for I. By the symmetry of the problem, and given \( A7(ii) \) and \( A8(ii) \), V will choose \( y = y^* \). Hence, the unique equilibrium outcome is \((x^*, y^*)\).

NdN: Consider I’s problem, assuming that V chooses \( y = y^* \). Then, I faces expected costs
\[
C^I(x^*; y^*) + L^I(x^*, y^*)
\]
by choosing \( x^* \), and
\[
C^I(x; y^*) + L^I(x, y^*) + L^V(x, y^*)
\]
by choosing \( x < x^* \). Given \( A5(i) \), I will choose \( x^* \). Now suppose that V chooses \( y < y^* \); then, I faces costs \( C^I(x^*; y) \) by choosing \( x = x^* \), and \( C^I(x; y) + L^I(x, y) \) by choosing \( x < x^* \). Given \( A8(i) \), I will choose \( x = x^* \). Thus, \( x = x^* \) is a dominant strategy for I. By the symmetry of the problem, and given \( A5(ii) \) and \( A8(ii) \), V will choose \( y = y^* \). Hence, the unique equilibrium outcome is \((x^*, y^*)\).

CN: Consider I’s problem, assuming that V chooses \( y = y^* \). Then, I faces expected costs
\[
C^I(x^*; y^*) + L^I(x^*, y^*)
\]
by choosing \( x^* \), and
\[
C^I(x; y^*) + L^I(x, y^*) + L^V(x, y^*)
\]
by choosing \( x < x^* \). Given \( A5(i) \), I will choose \( x^* \). Now suppose that V chooses \( y < y^* \); then, I faces costs \( C^I(x^*; y) \) by choosing \( x = x^* \), and \( \alpha[L^I(x; y) + L^V(x, y)] \) by choosing \( x < x^* \). Given \( A9(i) \), I will choose \( x = x^* \). Thus, \( x = x^* \) is a dominant strategy for I. By the symmetry of the problem, and given \( A5(ii) \) and \( A9(ii) \), V will choose \( y = y^* \). Hence, the unique equilibrium outcome is \((x^*, y^*)\).
Figure 1. Marginal Costs and Benefits of Precaution when $MC = MC(x)$

$\begin{align*}
\text{Figure 2. Marginal Costs and Benefits of Precaution when } MC &= MC(x,y) \\
\text{Figure 1. Marginal Costs and Benefits of Precaution when } MC &= MC(x)
\end{align*}$
<table>
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<td>1) Unilateral harm/independent costs</td>
<td>Shavell (1987, pp. 36f)</td>
<td>$C^I(x) + C^V(y) + L^V(x, y)$</td>
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<td>2) Bilateral harm/independent costs</td>
<td>Arlen (1990a)</td>
<td>$C^I(x) + C^V(y) + L^I(x, y) + L^V(x, y)$</td>
<td>*</td>
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<td>3) Unilateral harm/I affects V’s costs</td>
<td>Dharmapala et al. (2001)</td>
<td>$C^I(x) + C^V(y; x) + L^V(x, y)$</td>
<td>*</td>
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<tr>
<td>4) Unilateral harm/V affects I’s costs</td>
<td>Dharmapala et al. (2001)</td>
<td>$C^I(x; y) + C^V(y) + L^V(x, y)$</td>
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<td>5) Unilateral harm/interdependent costs</td>
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<td>*</td>
</tr>
</tbody>
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* indicates that the rules induces both I and V to undertake the socially optimal levels of care

Note that these results rely on the various assumptions stated in the text and in the cited works holding (e.g. A1-A4 for Model 5, and A5-A9 for Model 6)