Due date: To be collected at the beginning of class on Tuesday, October 27.

Note: The usual policy on cheating and plagiarism applies for this exercise.

1. Suppose the utility function of a representative young agent is:

\[ U = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \beta > 0. \]

Each agent receives \( y \) endowment when young and nothing when old. There is a fixed stock of money, \( M \), which is evenly distributed among the initial old.

a. Write down the representative agent’s budget constraint when young and when old, and derive the lifetime budget constraint.

b. Show that the demand for real money balance is

\[ \frac{\beta y}{1 + \beta}. \]

c. Derive the expressions for \( c_{1,t}^* \) and \( c_{2,t+2}^* \), where “*” denotes equilibrium.

d. Give an interpretation of the parameter \( \beta \), and describe the effects on \( c_{1,t}^* \) and \( c_{2,t+2}^* \) when \( \beta \) increases.

2. Consider an economy with a constant population of \( N = 100 \). Individuals are endowed with \( y = 20 \) units of the consumption good when young and nothing when old.

a. What is the equation for the feasible set of this economy? Portray the feasible set on a graph. With arbitrarily drawn indifference curves, illustrate the stationary combination of \( c_1 \) and \( c_2 \) that maximizes the utility of future generations.

b. Now look at a monetary equilibrium. Write down equations that represent the constraints on first- and second-period consumption for a typical individual. Combine these constraints into a lifetime budget constraint.

c. Suppose that the initial old are endowed with a total of \( M = 400 \) units of fiat money. What condition represents the clearing of the money market in an arbitrary period \( 0 \)? Use this condition to find the real rate of return of fiat money.

For the remaining parts of this exercise, suppose that preferences are such that individuals wish to hold real balances of money worth (note that this is the demand for fiat money derived with the utility function given in the example in the appendix to chapter 1)

\[ \frac{y}{1 + v_t / v_{t+1}} \] goods.

d. What is the value of money in period \( t \), \( v_t \)? Use the assumption regarding preferences and your answer in part c to find an exact numerical value. What is the price of the consumption good, \( p_t \)?

e. If the rate of population growth increased, what would happen to the rate of return of fiat money, the real demand for fiat money, the value of a unit of fiat money in the initial period, and the utility of the initial old? Explain your answers. Hint: Answer these questions in the order asked.
f. Suppose instead that the initial old were endowed with a total of 800 units of fiat money. How do your answers to part d change? Are the initial old better off with more units of fiat money?

3. Consider two economies, A and B. Both economies have the same population, supply of fiat money, and endowments. In each economy, the number of young people born in each period is constant at $N$ and the supply of fiat money is constant at $M$. Furthermore, each individual is endowed with $y$ units of the consumption good when young and zero when old. The only difference between the economies is with regard to preferences. Other things being equal, individuals in economy A have preferences that lean toward first-period consumption; individual preferences in economy B lean toward second-period consumption. We will also assume stationarity. More specifically, the lifetime budget constraints and typical indifference curves for individuals in the two economies are given in exercise 1.2 on page 28 of Champ and Freeman.

a. Will there be a difference in the rates of return of fiat money in the two economies? If so, which economy will have the higher rate of return of fiat money? Give an intuitive interpretation of your answer.

b. Will there be a difference in the value of money in the two economies? If so, which economy will have the higher value of money? Give an intuitive interpretation of your answer.