Modern attempts to erect a general theory of money and prices on Walrasian foundations have produced a model of economic phenomena that is suspiciously reminiscent of the classical theory of a barter economy. My purpose in this paper is to show that the conception of a money economy implicit in these constructions is empirically and analytically vacuous, and to propose an alternative microfoundation for the pure theory of a money economy.

I

For simplicity of exposition, I shall address my critique of the general equilibrium theory of money and prices to the classic statement presented in Don Patinkin's *Money, Interest and Prices*. Following Patinkin, suppose that we have to deal with an economy in which trading is rigidly synchronized within each of a series of discrete market periods. At the outset of every period, each transactor receives 'like manna from heaven' a collection of goods that may be consumed directly, or traded for a preferred collection of goods or for money at rates of exchange established by an independent market authority in accordance with prevailing conditions of market excess demand.

By hypothesis, market excess demands are defined in terms of individual demand functions for goods and money obtained as solutions to the decision problem:

\[ \text{Maximize } U_j(d_{1j}, \ldots, d_{nj}, M_j/P) \]

subject to

\[ \sum_{i=1}^{n} p_i(d_{ij} - s_{ij}) + \bar{M}_j - M_j = 0, \]

where \( U_j \) satisfies familiar continuity and curvature conditions, \( s_{ij} \) and \( d_{ij} \) represent initial and desired quantities of goods, \( M_j \) represent initial and desired quantities of fiat money, and \( P \) is a fixed-weight index of the money prices \( p_1, \ldots, p_n \). The empirical content of the theory thus derives ultimately from behavior restrictions implicit in equations 1 and 2. As it happens, the implications of the continuity and curvature conditions imposed on the utility functions 1 are exhausted in certain analytically subtle but empirically trivial restrictions involving the existence of demand functions and the continuity of their partial derivatives. The factual content of the theory depends very largely, therefore, on restrictions implicit in the budget equations 2. Accordingly, the main question that we have to answer in order to appraise the empirical significance of contemporary monetary theory is: *Do the budget equations 2 constitute an appropriate definition of choice alternatives in a money economy?*

That the appropriateness of the choice alternatives defined by 2 is open to serious question may be seen most easily by conducting a few simple conceptual experiments. First, consider an economy in which all transactors but one have a violent aversion to holding money balances. Starting from any first distribution of money balances, market trading over one or more periods will ultimately yield a situation in which the entire stock of money is held by a single individual. Changes in initial endowments of goods or in the stock of money will generate precisely the same qualitative effects in this model as would occur in a system where all transactors were willing to hold money balances in full equilibrium; hence the model differs in no essential respect from models discussed by Patinkin and other writers. But our model is so defined that, in equilibrium, money is not used in any exchange transaction. More pointedly, that fact that \( M_j = 0 \) for all but
one value of \( f \) in no way prevents any transaction from exerting an
influence on market excess demands for goods. For the goods
variables \( x \), enter the budget equations 2 in precisely the same
manner as the money variables \( M \), which is to say that goods
are indistinguishable from money as sources of effective demand.

Next consider a general Patinkin model in which all money
prices except that of labor services are free to vary. Suppose that
the economy, starting from a state of full equilibrium, experiences
a reduction in its stock of flat money (the result, let us say, of a
disastrous fire). The price level of goods other than labor will
decline; hence, real wages will rise and the demand for labor will
fall. When equilibrium is eventually re-established, therefore,
labor will be in excess supply. But the excess demand for all goods
except labor will be zero. By Walras’ law, therefore, the money
value of unsold labor will be positive and equal to the excess de-
mand for money-balances. If in this situation the quantity of labor
offered for sale by any transaction increases, the immediate effect
on demand for other goods will be the same as if the transaction
had experienced an increase in his stock of money. An auto-
nomous increase in unemployment will thus generate a rise in the
price level of other goods, a decline in the real wage rate, and so
an increase in employment and output! It can be shown, indeed,
that an increase in unsold stocks of any commodity the price of
which is fixed will, in a Patinkin-esque world, generate an increase
in the general price level and so, indirectly, a rise in sales of the
good whose price is fixed.\(^3\) Again, therefore, we arrive at a
conclusion that is offensive to our intuitive conception of the working
of a money economy, a conclusion that indicates that money
plays no distinctive role in economic activity.

The same result may be reached more directly by noticing that
the budget equations 2 admit as feasible trades every possible
combination of commodities traded in the economy; i.e., any
commodity, whether a good or money, can be offered directly in
trade for every other commodity. But an economy that admits of
this possibility clearly constitutes what any classical economist
would regard as a barter rather than a money economy (7). The fact

3. All of these conclusions are, of course, predicated on the assump-
tion that the economic system is stable; i.e., that prices adjust so that market
for all commodities whose prices are permitted to vary ultimately clear.

that flat money is included among the set of tradeable commodi-
ties is utterly irrelevant; the role of money in economic activity is
analytically indistinguishable from that of any other commodity.

II

The answer to our query about the appropriateness of the budget
constraints of established theory as a description of choice
alternatives in a money economy is negative; what presently
passes for a theory of a money economy is in truth descriptive of
a barter economy. I turn now to the task of reformulating accepted theory to reflect relevant restrictions on transaction behavior in a world where ‘money matters’.

The natural point of departure for a theory of monetary
phenomena is a precise distinction between money and nonmoney
commodities. In this connexion it is important to observe that
such a distinction is possible only if we assign a special role to
certain commodities as means of payment. For any commodity
may serve as a unit of account and standard of deferred payment:
and every asset is, by its very nature, a potential store of value.
If money is to be distinguished by the functions it performs,
therefore, it is to the medium of exchange function that we must
address our attention. The only difficulty is to express analytically
what is meant when we assert that a certain commodity serves as
a medium of exchange.

To resolve this difficulty, we proceed by associating with any
set of commodities \( C = (C_1, \ldots, C_n) \) an exchange relation, \( E \),
defined as a subset of the Cartesian product \( C \times C \) of the
commodity set \( C \); i.e. a set of ordered pairs of commodity elements
\( (C_i, C_j) \). We then say that a trade involving two commodities \( C_i \)
and \( C_j \) is feasible if and only if the pair \( (C_i, C_j) \) is an element of
the exchange relation, and we write \( C_i,E C_j \) (read ‘commodity \( i \)
can be traded directly for commodity \( j \)’) to indicate that this
condition is met.

The exchange relation is necessarily non-empty and reflexive, for
since any commodity available for trade can be held by the
individual who possesses it, the condition \( C_i, E C_j \) is satisfied
vacuously for all possible values of \( i \). Moreover, the exchange
relation is symmetric; for if \( C_i, E C_j \), then by the nature of trade it
must also be true that \( C_i ; EC_i \). In general, however, the exchange relation need not be transitive; that is to say, \( C_i ; EC_j \) and \( C_j ; EC_k \) may or may not entail \( C_i ; EC_k \). Transitivity of the exchange relation of an economy is, in fact, characteristic of just one class of economies, namely, barter economies. This follows directly from the definition of a barter economy as one in which any commodity may be offered directly in trade for every commodity; i.e., an economy for which \( C_i ; EC_j \) is true for all values of \( i \) and \( j \).

Now define as a money commodity any element \( C_i \) of \( C \) for which \( C_i ; EC_j \) is true for all values of \( j \); i.e., any commodity that can be traded directly for all other commodities. It then follows that a barter economy is one in which all commodities are money commodities. This characterization of a barter economy may seem paradoxical at first sight; but if one ponders the matter it becomes clear that the peculiar feature of a money as contrasted with a barter economy is precisely that some commodities in a money economy cannot be traded directly for all other commodities; i.e., some exchanges necessarily involve intermediate monetary transactions. More precisely, we now define a money economy as a system involving at least one money commodity but a non-transitive exchange relation. We note in passing that the simplest money economy must contain at least three commodities as illustrated in Table 1(b) (X indicates that \( C_i ; EC_j \); 0 that \( C_j ; FC_i \)). For if it contains fewer than three, then as indicated in Table 1(a), the reflexivity and symmetry of the exchange relation implies that all elements of \( C \times C \) are included in \( E \). It follows – in keeping with common sense – that every two-commodity economy is a barter economy. Similarly, the simplest money economy that admits simultaneously of certain forms of barter (i.e., direct trading of certain commodities none of which are money commodities) must contain at least four commodities, as illustrated in Table 1(c). In general, the exchange relation of a money economy may contain numerous barter subsets (trade credit, blocked currencies, credit cards, demand deposits, etc.). Such non-pure money economies (as I shall call them) seriously complicate the task of defining relevant choice alternatives for transactors. Accordingly, I shall restrict attention in the argument that follows to pure money economies in which one and only one commodity can be traded directly for any other commodity.

So much for the distinction between money and other commodities and for the characterization of money as contrasted with barter economies. A commodity is regarded as money for our purposes if and only if it can be traded directly for all other commodities in the economy. Correspondingly, a money economy is one in which not all commodities are money. It should perhaps be observed that the feasibility of particular trades is tacitly assumed to be determined by institutional and environmental rather than economic considerations. That is to say, the exchange relation of an economy either does or does not assign a special role to certain commodities as money. The distinction between money and other commodities is thus a matter not of degree but of kind. One might express this thought more elliptically by saying that money is traded in all markets in a money economy, or, alternatively, by saying that transaction costs are in some for any market exchange that does not involve the offer of money as a means of payment.

III

Having enunciated sharp definitions of relevant primitive terms, it is now a straightforward matter to characterize the choice alternatives open to transactors in a pure money economy. We begin by recalling that the peculiar feature of a money economy is that some commodities (in the present context, all but one) are denied a role as potential or actual means of payment. To state the same idea as an aphorism: Money buys goods and goods buy money (2).
Contemporary Theory: Neoclassical Equilibrium Analysis

Money; but goods do not buy goods. This restriction is – or ought to be – the central theme of the theory of a money economy. The task of reformulating microeconomic analysis to accommodate those aspects of experience that are commonly supposed to distinguish a money from a barter economy consists, indeed, of little more than an elaboration of the implications of this one restriction.

Our aphorism automatically rules out the standard budget constraints of neoclassical equilibrium analysis as accurate descriptions of planning alternatives open to transactors in a money economy. For, as remarked earlier, the familiar budget constraint effectively admits as feasible trades all pairwise combinations of commodities that are traded in the economy. In sharp contrast, choice alternatives in a money economy must be so defined as to satisfy the requirement that money be offered or demanded as one of the commodities entering into every trade. Analytically, what this entails is a clear separation between goods demanded for purchase (offers to sell money) and goods offered for sale (offers to buy money). This condition may be met most easily by dichotomizing the budget constraint into two branches, the first representing a constraint on money expenditure, the second representing a constraint on money income. Symbolically, we have

\[ \sum_{i} P_i x_{ij} + M_j - M'_j = 0, \quad x_{ij} = d_{ij} - s_{ij}, \text{where } x_{ij} \geq 0 \]

as the expenditure constraint, and

\[ \sum_{i} P_i x_{ij} + m_j = 0, \quad \text{where } x_{ij} < 0 \]

as the income constraint, where \( m_j \) represents desired “intra-period” receipts of money income and all other symbols are defined precisely as before.

The expenditure constraint asserts that all (net) purchase offers must be backed by a readiness to supply money in exchange. Note that the expenditure constraint cannot be satisfied for non-negative values of all \( x_{ij} \) unless \( M_j - M'_j \) is non-positive (i.e., \( 0 \leq M_j \leq M'_j \)). Thus \( M \) corresponds to what is commonly referred to as demand for precautionary money balances; i.e., total initial cash balances less prospective (gross) depletions of cash balances for currently scheduled purchases of goods. It follows that the total value of goods demanded cannot in any circumstances exceed the amount of money held by the transactor at the outset of the period. Our definition of choice alternatives thereby captures the essential meaning of the traditional (but curiously non-modern) contention that demand in a money economy is effective only if it involves a combination of desire with money purchasing power.

The income constraint asserts that all (net) sale offers involve a demand for just one other commodity, namely money, in exchange. Thus \( m_j \) corresponds to what is commonly referred to as demand for transactions balances (to replace cash currently scheduled for disbursement from initial holdings of money balances). For convenient reference, I shall refer henceforth to \( m_j \) as the income demand for money, to \( M \) as the reservation demand for money.

Such a separation of income and reservation demands has no place in accepted equilibrium models, for these models invariably contain a budget constraint that permits but does not require that offers to purchase goods be backed by willingness to give, and that offers to sell goods be accompanied by willingness to receive units of money in exchange. The omission of these restrictions from contemporary monetary theory is a natural consequence of the tacit presupposition, appropriate in a world of transaction or recontract, that money is just one of many commodities that may be bartered directly for other commodities.

Given the budget restraints 3 and 4, we may write the utility function of a typical transactor as

\[ U_j(d_{1j}, \ldots, d_{nj}, M_j, m_j, P, m_j, P) \]

and proceed as usual to define individual demand functions, for goods, reserves, balances and money income as solutions to the problem of maximizing 5 subject to the expenditure and income constraints 3 and 4. The inequality conditions that determine whether a given net demand \( x_{ij} \) enters the expenditure or the income constraint preclude us from stating precise conditions that will guarantee the existence of the required solutions.

5. The income constraint cannot be satisfied for non-positive values of \( x_{ij} \); like the variable \( M_j \), therefore, \( m_j \) is, in general, positive.
Supposing that the decision problem has a solution, however, we arrive at individual demand and excess-demand functions that differ in crucial respects from those of established theory.

As in established theory, so in our model the demand functions for goods and for real money income and real reservation balances are homogeneous of degree zero in money prices and initial money balances. In contrast with established theory, however, substitution effects of changes in price are asymmetrical unless both commodities are either offered for sale or demanded for purchase. Unlike established theory, moreover, a change in initial money balances has no 'income' effect on goods offered for sale. More significantly, changes in initial endowments of goods have no 'income' effect on commodities that are demanded for purchase; i.e., supply of goods does not create demand for other goods. All of these results are obvious consequences of dichotomizing budget constraints into separate expenditure and income branches.

As in established theory, the money value of the sum of all excess demands, including the excess demand for reservation balances and for money income, is identically zero; hence a proposition analogous to what has come to be known as Walras' law applies to transactors in a money as well as to transactors in a barter economy. As noted earlier, however, in our model the money value of the sum of gross demands for goods and for reservation balances is identically equal to initial cash balances. Demand (and excess-demand) functions in a money economy are thus subject to much more severe restrictions than are those of a barter economy—the latter category being interpreted to include all neo-Walrasian models of money and prices.

A full statement of the implications of my proposed microfoundation for monetary theory cannot be given here. For present purposes, it is sufficient to observe that the results given above ensure that the response of transactors to changes in prices or in initial endowments of goods or money will differ qualitatively from findings suggested by established theory. Correspondingly, the response of market prices and quantities traded to changes in

6. Certain qualifications are in order if the initial solution value of \( x_{1i} = 0 \), for in this case changes in \( M_2 \) or \( x_{ij} \) determine which branch of the budget constraint the terminal solution value of \( x_{1i} \) enters.